System 83 Leo – two planets’ orbit of one star: mapping possibilities for the system

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Abstract

Our focus is on binary stellar systems that host extrasolar planets which orbit one of the stars (S-type) (Dvorak 1986). We have investigated the motion of planets in the case of the three-body problem (Plávalová & Solovaya 2013, AJ, 146, 108). We can completely solve the three body problem given the initial conditions of: (1) a planet in a binary system revolves around one of the components (parent star); (2) the distance between the star’s components is greater than that between the parent star and the orbiting planet (ratio of the semi-major axes is a small parameter); and (3) the mass of the planet is less than the mass of either star, but is not negligible. The solution of the system was obtained and qualitative analysis of the motion was made. We have applied this theory to system 83 Leo (ADS8162), whose B-component has two orbiting planets, calculating their unknown angular orbital elements; inclination and ascending node. Using this new data, we have determined if this system could be stable via numerical calculation. We have discussed the possible construction of systems like this one.

1 Introduction

We targeted binary stellar systems which are hosting extrasolar planets. We have focused on an S-type orbit (Dvorak 1986), where an extrasolar planet orbits one of the stars (parent star) and targeted its motion. We considered the ratio of the semi-major axis of the orbits of a planet and the distant star as a small parameter. The mass of the planet is much smaller than the mass of the stars, but is not negligible. The motion is considered in the Jacobian coordinate system and the invariable plane is taken as the reference plane. For a description of the evolution we have used the Delaunay canonical elements $L_i, G_i, H_i, l_i, g_i$ which can be expressed through the Keplerian elements as:

$$
\begin{align*}
L_i &= \beta_i \sqrt{a_i}, \\
G_i &= L_i \sqrt{1 - e_i^2}, \\
H_i &= G_i \cos I_i, \\
l_i &= M_i, \\
g_i &= \omega_i, \\
h_i &= \Omega_i,
\end{align*}
$$

(1)

where $i = 1$ is for the planet’s orbit, and $i = 2$ is for the distant star’s orbit, and other variables have the usual meaning, i.e. $m_0, m_2$ – mass of the stars, $m_1$ – mass of the planet, $\beta_i$ – the coefficients depend on their masses, $a_i$ –the semi-major axis, $e_i$ – the eccentricity, $M_i$ – the mean anomaly, $I_i, \omega_i, \Omega_i$ – are the angular variables for the observation plane, and $g_i$ – the argument of the pericenter according to the invariable plane (perpendicular to the angular momentum of the system). The eccentricities of the star’s and planet’s orbits can have any value from $0 < e_i < 1$. The solution of the task using the Hamiltonian without short-periodic terms, was obtained in hyper elliptic integrals using the Hamilton-Jacobi method (Orlov & Solovaya 1988). The short-periodic terms are small
and have no significant influence on the dynamic evolution of the system, the values of which are less than ±0.10^{-3} which is beyond the precision capabilities for the observations.

\[
F = \frac{\gamma_1}{2L_1^2} + \frac{\gamma_2}{2L_2^2} - \frac{1}{16} \gamma_3 \frac{L_1^4}{L_2^2 G_2^2} \left[(1 - 3q^2) (5 - 3q^2) - 15 \left(1 - q^2 \right) \left(1 - \eta^2 \right) \cos(2g_1) \right],
\]

(2)

where the coefficients \( \gamma_1, \gamma_2, \) and \( \gamma_3 \) depend on their masses, \( q \) is the cosine of the angle between the plane of the planet’s orbit and plane of the distant star’s orbit and \( \eta = \sqrt{1 - e^2} \). The eccentricity \( e \) of the distant star is constant but the eccentricity \( e_1 \) of the planet’s orbit can change in a long interval.

According to the Hamiltonian (2) solution, under the integral in the denominator is the square root from the polynomial of the fifth order. This polynomial can be presented as the product of the two polynomials; the second and the third orders, where

\[
f_3(\xi) = \xi^3 - 2\left(\xi^2 + 3G_2^3\right)\xi + \left(\xi^2 - G_2^3\right)^2 + \frac{2}{3} \left(10 + A_3\right) G_2^2,
\]

(3)

and

\[
f_5(\xi) = \xi^5 - 2\left(\xi^2 + \xi^2 + \frac{5}{4}\right) \xi^2 + \left[\frac{5}{2} \left(\xi^2 + G_2^3\right) + \left(\xi^2 - G_2^3\right)^2 - \frac{1}{6} G_2^2 (10 + A_1)\right] \xi - \frac{5}{4} \left(\xi^3 - G_2^3\right)^2.
\]

Where \( \epsilon \) is the constant of the angular momentum of the system and \( A_3 \) is a parameter and,

\[
A_3 = 2 - 6\eta^2 q^2 - 6 \left(1 - \eta^2\right) - 2 - 5 \left(1 - q^2\right) \sin^2 g_1 \quad \text{and} \quad \gamma = \frac{\epsilon}{A_3}.
\]

(4)

When in the pericenter \( r_p = a_1(1 - e_{\text{max}}) \), the planet’s approach to the Roche limit then the parent star’s tidal forces to destroy the planet. For the calculation of the Roche limit \( d_R \), we used the equation published by Eggleton (1983):

\[
d_R = \frac{0.49 \mu^3}{0.6 \mu^2 + \ln \left(1 + \mu^2\right)},
\]

(5)

where \( \mu = m_1 / m_0 \). The eccentricity of an extrasolar planet which reaches the Roche limit is \( e_{1R} = 1 - d_R / a_1 \) (critical eccentricity). The determination of the regions of the motion of a planet are possible when the roots of the equations \( f_3(\xi) = 0, f_5(\xi) = 0 \) are found and the signs of the functions defined (Plávalová & Solovaya 2013). If the difference between the smaller root of a quadratic equation \( f_3(\xi) \) and the smallest root of the cubic equation \( f_5(\xi) \) is minimal, the motion of the planet would be stable. This means that \( e_1 < e_{1R} \). When \( e_{\text{max}} \) is close to or reaches \( e_{1R} \), a planet’s orbit can not stable.

Table 1: Initial orbital elements of the system 83 Leo.

<table>
<thead>
<tr>
<th></th>
<th>83 Leo Bb</th>
<th>83 Leo Bc</th>
<th>83 Leo A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum mass ( (M_{\text{Jup}}) )</td>
<td>0.109</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td>Mass ( (M_{\odot}) )</td>
<td>0.83 (parent star)</td>
<td>1.01</td>
<td></td>
</tr>
<tr>
<td>Semi-major axis (au)</td>
<td>0.1231</td>
<td>5.4</td>
<td>367</td>
</tr>
<tr>
<td>Roche limit ( d_R ) (au)</td>
<td>0.0244</td>
<td>0.0362</td>
<td></td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.254</td>
<td>0.106</td>
<td>0.46</td>
</tr>
<tr>
<td>Critical eccentricity ( e_{1R} )</td>
<td>0.8019</td>
<td>0.9933</td>
<td></td>
</tr>
<tr>
<td>Period (day)</td>
<td>17.0431</td>
<td>4970.0</td>
<td>5176</td>
</tr>
<tr>
<td>Argument of perigee (°)</td>
<td>219</td>
<td>38</td>
<td>112.9</td>
</tr>
<tr>
<td>Ascending node (°)</td>
<td>164.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inclination (°)</td>
<td>126.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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2 System 83 Leo

To investigate the evolution of the planet’s orbit and the possible conditions of the stability, it is necessary to know the six Keplerian elements for the planet and for the distant star. The existing observational techniques do not allow us to define all of these elements unambiguously. In the catalogue of extrasolar planets, the data for the longitude of the ascending node and the inclination value are generally absent. Our theory allows us to calculate the possible values for these unknown elements for which a planet’s motion is stable over an astronomically long-time interval.

We targeted system 83 Leo (ADS8162) which contains an A component spectral class G9IV-V with a mass of $1.01 M_{\text{Sun}}$ and a B component spectral class K2V with a mass of $0.86 M_{\text{Sun}}$. In our calculation we used orbital elements published by [Hopmann 1960] (Tab. 1). We calculated the value of the semi-major axis using the third Kepler law. Extrasolar planet 83 Leo Bb (HD99492 Bb) with a mass of $0.109 M_{\text{Jup}}$ and the taxonomy minimal code NG (Plávalová 2012) and 83 Leo Bc (HD99492 Bc) with a mass of $0.36 M_{\text{Jup}}$ and the taxonomy minimal code NF, are orbiting the B component of the system. We have used the values for the orbital elements for 83 Leo Bb published by [Butler et al. 2006] and for 83 Leo Bc elements published by [Meschiari et al. 2011] in our calculations (Tab. 1). We calculated the values of the Roche limit $d_R$ and the critical eccentricity $e_{1R}$ for both planets (Tab. 1).

![Graph](Leo83Bce1maxdashed.pdf)

**Figure 1:** 83 Leo Bb and 83 Leo Bc: The evolution of the maximum and minimum value of the planet’s eccentricity $e_1$ vs. the ascending node of the planet $\Omega_1$ for which $I_1 = 54^\circ$ – blue line, and $I_1 = 126^\circ$ – green line. The maximum values of the planet’s eccentricity are plotted with solid lines and the minimum values with the dotted lines. The critical eccentricity $e_{1R}$ is plotted with a red line.

<table>
<thead>
<tr>
<th></th>
<th>83 Leo Bb</th>
<th>83 Leo Bc</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HD99492 Bb</td>
<td>HD99492 Bc</td>
</tr>
<tr>
<td>Minimum mass ($M_{\text{Jup}}$)</td>
<td>0.135±0.007</td>
<td>0.451±0.077</td>
</tr>
<tr>
<td>Semi-major axis (au)</td>
<td>0.1231</td>
<td>5.4</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.254</td>
<td>0.106</td>
</tr>
<tr>
<td>Argument of perigee ($^\circ$)</td>
<td>219</td>
<td>38</td>
</tr>
<tr>
<td>Ascending node ($^\circ$)</td>
<td>165±5</td>
<td>164±13</td>
</tr>
<tr>
<td>Inclination ($^\circ$)</td>
<td>126±4</td>
<td>127±10</td>
</tr>
</tbody>
</table>

Table 2: Our proposal for possible orbital elements for planets 83 Leo Bb and 83 Leo Bc.

The values for the ascending nodes and these planets’ inclinations are an unknown. We have made some calculations by our analytical theory for each planet separately, because the distance between the planets is more than 5 au and the masses of both planets are less than half of Jupiter’s mass. We can then consider the planet’s
3 Comparison of theoretical results with numerical integration

We compared our results obtained from analytical theory with those from the numerical integration of the equation of motion of the planets. We used N-body integrator Mercury (Chambers 1999) and the Everhart integration method (Everhart 1985). The equations of the motion of the system were numerically integrated through 10^5 yrs. We calculated the whole system (two stars and two planets) in one calculation, including the masses of the planets too. We used the initial value of the planet’s inclination \( I_1 = 53^\circ \) and the planet’s ascending node \( \Omega_1 = 110^\circ \), \( \Omega_1 = 220^\circ \), and then \( \Omega_1 = 344^\circ \). For an illustration, the evolution of the eccentricity \( e_1 \) and the pericenter distance \( r_p = a_1 (1 - e_1) \) of the planets 83 Leo Bb and 83 Leo Bc are presented in Fig. 2.

The results obtained by analytical theory and by numerical integration compare quite well. For example, for a system where both planets have \( I_1 = 53^\circ \) and \( \Omega_1 = 220^\circ \) the results from analytical theory show an unstable system and the results from numerical integration confirmed that. The value of the eccentricity of 83 Leo Bb, reaches critical eccentricity after 10 million years, exceeding the Roche limit, and the parent star’s tidal forces could destroy it. Moreover, the variation in eccentricity of 83 Leo Bc over a period less than 100 kyrs, is very broad; from 0.001 to values of critical eccentricity. After the first 150 kyrs, the eccentricity reaches critical eccentricity. Contrariwise, for a system where both planets have \( I_1 = 53^\circ \) and \( \Omega_1 = 344^\circ \), the results from analytical theory show a stable system which is also confirmed by numerical integration. The time variations in eccentricity for both planets are minimal. These relatively quick results confirm the favourable benefit of using analytical theory where we are able to calculate the values which describe a stable system directly.

4 Conclusion

Here we have investigated the motion of extrasolar planets 83 Leo Bb and 83 Leo Bc orbiting one of the stars in a stellar binary system. We have used the analytical theory to calculate a range of values for this element within which the planet’s orbit would be stable for an astronomically long time scale. We found the range of the values of 83 Leo Bb to be: \( I_1 (122^\circ, 130^\circ) \) and \( \Omega_1 (160^\circ, 170^\circ) \) (or \( I_1 (50^\circ, 58^\circ) \) and \( \Omega_1 (340^\circ, 350^\circ) \)) and the range of the values of 83 Leo Bc to be: \( I_1 (117^\circ, 137^\circ) \) and \( \Omega_1 (151^\circ, 177^\circ) \) (or \( I_1 (43^\circ, 63^\circ) \) and \( \Omega_1 (331^\circ, 357^\circ) \)). The two variants of the results show contrary orientation of the orbits. Using the calculated inclination of both planets, we received the planets mass of 83 Leo Bb to be \( 0.135 \pm 0.007 M_{\text{Jup}} \) and 83 Leo Bc to be \( 0.451 \pm 0.047 M_{\text{Jup}} \).

We showed the results obtained by analytical theory and by numerical integration compare quite well. However, there is a significant benefit to using analytical theory as it is a speedier process, where we are able to calculate the values which describe a stable system directly.

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Figure 2: The evolution of the planet’s eccentricity $e_1$ and the pericenter distance $r_p = a_1(1 - e_1)$ (au) over $10^7$ yrs ($t=10^6$ years). The curves are the result of the numerical integration. For $I_1 = 53^\circ$, and $\Omega_1 = 110^\circ$ (brown), $\Omega_1 = 220^\circ$ (green), and $\Omega_1 = 344^\circ$ (blue).

References


Hopmann, J. 1960, Mitteilungen der Universitaets-Sternwarte Wien, 10, 155


Plávalová, E. 2012, Astrobiology, 12, 361