The secrets revealed by multi-planet systems

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Monash & Geneva
Outline

What can we learn from planets with companions that we can’t learn from single planets?

• Transiting hot jupiters with eccentric companions (eg HAT-P-13)

• Non-transiting resonant systems (eg GJ 876)

• Transiting planets with TTVs
1995

51 Peg

- Migrated in? (Lin, Bodenheimer & Richardson 1996)

- Scattered in? (Rasio & Ford 1996)
2015
Lonely Hot Jupiters

- Migrated in?

- Scattered in?

Huge amount of theoretical work but no conclusions yet...

- Kozai
- Chaotic scattering
- + tides
- Influence of gas giants on planet formation
- Effect of cluster environment
- etc

More constraints are needed from observations - parameter space is rapidly filling in from many observational techniques...
Period ratio distribution for pairs of giant planets
- both planets have mass $> 0.3 \, M_J$

from exoplanets.eu
includes adjacent pairs when $n>2$
Small period ratios: comparison with Kepler pairs

red = both planets have mass > 0.3 M_J
Period ratio as a function of inner period
Systems with short-period planets - lonely?

HD187123: $p_2/p_1 = 1230$

HD217107: $p_2/p_1 = 591$

Sunday, 22 November 15
Transiting short-period planets with distant eccentric companions give us the opportunity to probe the **interior structure** of the short-period planet.

<table>
<thead>
<tr>
<th></th>
<th>P₁ (days)</th>
<th>P₂/P₁</th>
<th>Teq</th>
<th>c₁(eq)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HAT-P-13</td>
<td>2.9</td>
<td>150</td>
<td>ok</td>
<td>0.02</td>
</tr>
<tr>
<td>WASP-41</td>
<td>3.1</td>
<td>138</td>
<td>ok</td>
<td>0.001</td>
</tr>
<tr>
<td>Kepler-424</td>
<td>3.3</td>
<td>68</td>
<td>ok</td>
<td>0.05 ec</td>
</tr>
<tr>
<td>WASP-47</td>
<td>4.2</td>
<td>138</td>
<td>too long?</td>
<td>(0.003 +/- 0.003)</td>
</tr>
<tr>
<td>HAT-P-44</td>
<td>4.3</td>
<td>51</td>
<td>too long?</td>
<td>(0.07 +/- 0.07)</td>
</tr>
<tr>
<td>HAT-P-46</td>
<td>4.5</td>
<td>17</td>
<td>too long</td>
<td>(0.12 +/- 0.12)</td>
</tr>
</tbody>
</table>
Probing the internal structure of short-period planets

Fixed-point theory of tidal evolution of planets with companions
(Mardling 2007, 2010, Wu & Goldreich 2001)

\[ e_b > 0 \]

\[ 3 \tau_{\text{circ}} \]

Need \( 3 \tau_{\text{circ}} \) to be shorter than the age of the system
Probing the internal structure of short-period transiting planets

\[ e_b^{(eq)} = \frac{(5/4)(a_b/a_c) e_c \varepsilon_c^{-2}}{|1 - \sqrt{a_b/a_c (m_b/m_c)} + \gamma|} \]

independent of \( Q_b \)

(Mardling 2007)

\[ \varepsilon_c = \sqrt{1 - e_c^2}, \quad \gamma = \gamma^{GR} + \gamma^{tidal \ bulge} \]

proportional to planet Love number

\(- \quad e_b^{(eq)} > 0 \)
Batygin et al (2009) realized that an accurate measurement of $e_b^{(eq)}$ allows one to probe the internal structure of the transiting planet via the Love number. eg. Does the planet have a core?
Equilibrium eccentricity substantial if:

- $m_c/m_b$ large (there are interesting exceptions)
- $a_b/a_c$ not too small
- $e_c$ large

**HAT-P-13:** $m_c/m_b \approx 18$, $e_c = 0.7$, $a_b/a_c = 0.03$

$e_b \approx 0.02$ 

Batygin et al 2009, Mardling 2010

consistent with observed value
Probing the architecture of non-transiting systems

**GJ 876**: 4 planets, 2 in 2:1 resonance

The strong non-Keplerian planet-planet interactions allows one to determine all orbital parameters of resonant pair including inclination from radial velocity data (Correia et al 2010)

**Ups And**: 4 planets including 2 with masses > 10 M\textsubscript{J}. Large masses allow measurement of inclinations using RV + astrometry (McArthur et al 2010)
Transit Timing Variations

Kepler: \textit{zillions} of planet radii, only a few masses :-( :-(

2600 systems show TTVs

All those TTVs contain information about the planet masses and orbital parameters

Radii + masses = \textit{planetology}

- how does one extract this information efficiently and accurately????
Transit Timing Variations

The time of mid-transit of (truly) single transiting planets is perfectly periodic.

If another planet resides in the system, this is no longer true for potentially three reasons:

1. Barycentric motion
2. Light-travel time
3. Planet-planet interaction
Transit Timing Variations

1. Barycentric motion: transits of the innermost planet

Barycentric motion does **NOT** produce measurable TTVs for the innermost planet.
Transit Timing Variations

1. Barycentric motion: transits of the outmost planet

Barycentric motion **DOES** contribute to the TTVs of the outermost planet.
Transit Timing Variations

2. Light-travel time

Changes in the light travel time due to barycentric motion do not produce measurable TTVs for planetary systems (but do for triple stars).
Transit Timing Variations

3. Planet-planet interaction

TTVs are a result of short-term variations in the transiting planet’s

(a) eccentricity
(b) orbital period
(c) longitude of periastron
(d) mean longitude
Transit Timing Variations

3. Planet-planet interaction

Near-resonant and resonant systems of planets tend to produce the largest TTVs because these variations add coherently.
What about systems far from resonance?

 Kepler-117b: a system of two transiting planets with period ratio 2.7.

 $P_1=18.7$ days - TTV amplitude proportional to period
Periodogram of TTVs from Bruno, Almenara, Barros, Santerne, Diaz, Deleuil, Damiani, Bonomo, Boisse, Bouchy, Hebrard, Montagnier, (~all OHP 2015 conference participants), 2014
Kepler-117b: a system far from resonance

TTVs of inner planet, folded at the outer period (Bruno et al 2014)
Kepler-117b: a system far from resonance

Challenge: find the analytical form of the folded curve...

If we can do that, we can match it to a least-squares fit and solve for the masses and elements.
Fourier transform (frequency-o-gram) of TTV data

The sampling frequency is once per inner orbit so the Nyquist frequency is half the inner orbital frequency.
Fourier transform (frequency-o-gram) of TTV data

Peaks at higher frequencies are aliases of the peaks below the Nyquist cutoff.
What is the physical origin of these peaks?

Variations in the eccentricity of the inner planet:

Although the system is "far" from exact commensurability, there is still some coherent behaviour.
What is the physical origin of these peaks?

Variations in the period of the inner planet:

The inner period varies with a frequency shorter than the Nyquist frequency.
where is the real power?

analysis of an N-body integration

Normalized power

Normalized power

Normalized power

 normalized power

(normalized power)
Information about the system is sampled once per inner period and so $P_c/P_b$ per outer period.

Hence the time resolution of the dynamics is via the outer orbit.
$t = T_0$
\[ t = T_0 + P_b \]

\[ \theta = \frac{2\pi}{(\text{period ratio})} \]
\[ t = T_0 + 2P_b \]
\[ t = T_0 + 3P_b \]
$t = T_0 + 4P_b$
$t = T_0 + 5P_b$
\[ t = T_0 + 6P_b \]
TTVs folded at outer period
We can use the machinery of celestial mechanics to derive a Fourier expression for the TTVs. (Also see Deck & Agol 2015)

Such a formula must reflect the time sampling of the outer orbit:

\[ TTV(j) = P_b(T_0) \sum_{n=1}^{n_{max}} A_n \sin[n(j\theta) - \beta_n] \]

\[ \theta = 2\pi/(\text{period ratio}) \]

The TTV amplitudes \( A_n \) and phases \( \beta_n \) are functions of \( m_c/m_*, e_b, e_c, \omega_b, \omega_c, \) and \( \lambda_c(T_0) \)

- all the information we wish to know about the system
Procedure to solve for perturber mass and elements

• For Kepler-117, there are three dominant harmonics.

• Each harmonic has an amplitude and a phase.

• Thus we have 6 equations for 6 unknowns. A correct solution should `predict’ amplitudes and phases of other harmonics

Such a technique is zillions of times faster than N-body...
1. Least-squares Fourier fit of data (folded at outer period)

<table>
<thead>
<tr>
<th>n'</th>
<th>amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.5</td>
</tr>
<tr>
<td>2</td>
<td>7.2</td>
</tr>
<tr>
<td>3</td>
<td>6.0</td>
</tr>
<tr>
<td>4</td>
<td>0.9</td>
</tr>
</tbody>
</table>
2. Match analytic and least-squares amplitudes and phases and solve for perturber mass and elements

A first guess for this procedure is given by simplified version of equations

\[ \frac{m_c}{m_*}, \quad e_b, \quad e_c, \quad \omega_b, \quad \omega_c, \quad \lambda_c(T_0) \]
3. Use those elements and mass to run N-body as a check
<table>
<thead>
<tr>
<th></th>
<th>Bruno et al</th>
<th>this analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_c (M_J)$</td>
<td>$1.84 \pm 0.18$</td>
<td>1.73</td>
</tr>
<tr>
<td>$e_b$</td>
<td>$0.049 \pm 0.006$</td>
<td>0.032</td>
</tr>
<tr>
<td>$e_c$</td>
<td>$0.032 \pm 0.003$</td>
<td>0.039</td>
</tr>
</tbody>
</table>
A system close to resonance

We can tell the system is near resonance because of the long period of variation of the TTVs.

But which resonance? We don’t know the period ratio...
A system close to resonance

1. The Fourier transform (Lomb-Scargle) of the signal gives possible period ratios.

\[ \frac{\omega}{n_b} = n - \frac{n'}{\sigma} \]

period ratio could be
2.04, 3.06, 4.08... 1.515...
Try folding signal with period ratio 2.0489

<table>
<thead>
<tr>
<th>n’</th>
<th>amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>100.0</td>
</tr>
<tr>
<td>3</td>
<td>3.6</td>
</tr>
<tr>
<td>4</td>
<td>12.6</td>
</tr>
<tr>
<td>5</td>
<td>0.5</td>
</tr>
<tr>
<td>6</td>
<td>5.4</td>
</tr>
</tbody>
</table>
Fitting first three harmonics gives

A system close to resonance

Fits data but not consistent with N-body
A system close to resonance

Fitting $n' = 1, 2, 4$ gives

Fits data but not consistent with N-body
Try folding signal with period ratio 3.073

<table>
<thead>
<tr>
<th>n’</th>
<th>amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.4</td>
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<tr>
<td>2</td>
<td>0.8</td>
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<tr>
<td>3</td>
<td>100.2</td>
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<tr>
<td>4</td>
<td>2.5</td>
</tr>
<tr>
<td>5</td>
<td>2.3</td>
</tr>
<tr>
<td>6</td>
<td>12.6</td>
</tr>
</tbody>
</table>

Fit using n’=1,3,6
A system close to resonance: period ratio = 3.07

Fitting \( n' = 1, 3, 6 \) gives

Solution consistent with N-body

<table>
<thead>
<tr>
<th>( m_c ) (M_J)</th>
<th>0.24</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e_b )</td>
<td>0.02</td>
</tr>
<tr>
<td>( e_c )</td>
<td>0.14</td>
</tr>
</tbody>
</table>
A system close to resonance: period ratio = 1.89

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_c \ (M_J)$</td>
<td>0.03</td>
</tr>
<tr>
<td>$e_b$</td>
<td>0.02</td>
</tr>
<tr>
<td>$e_c$</td>
<td>0.04</td>
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