Image Reconstruction for Sparse Aperture Masking and Kernel-phase techniques

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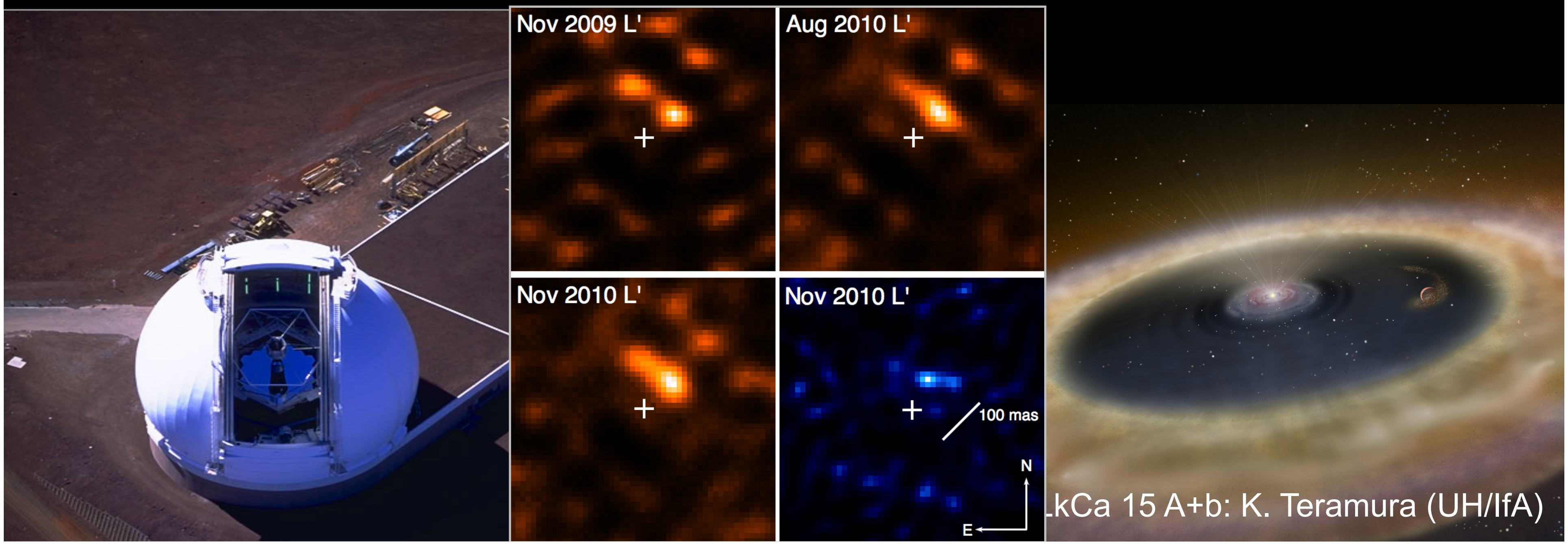


Outline

1. Examples of SAM image reconstruction (including hybrid imaging)
2. Aperture Masking and Kernel-phase: are these really different techniques to LBOI?
3. Symmetric and Anti-symmetric image components.

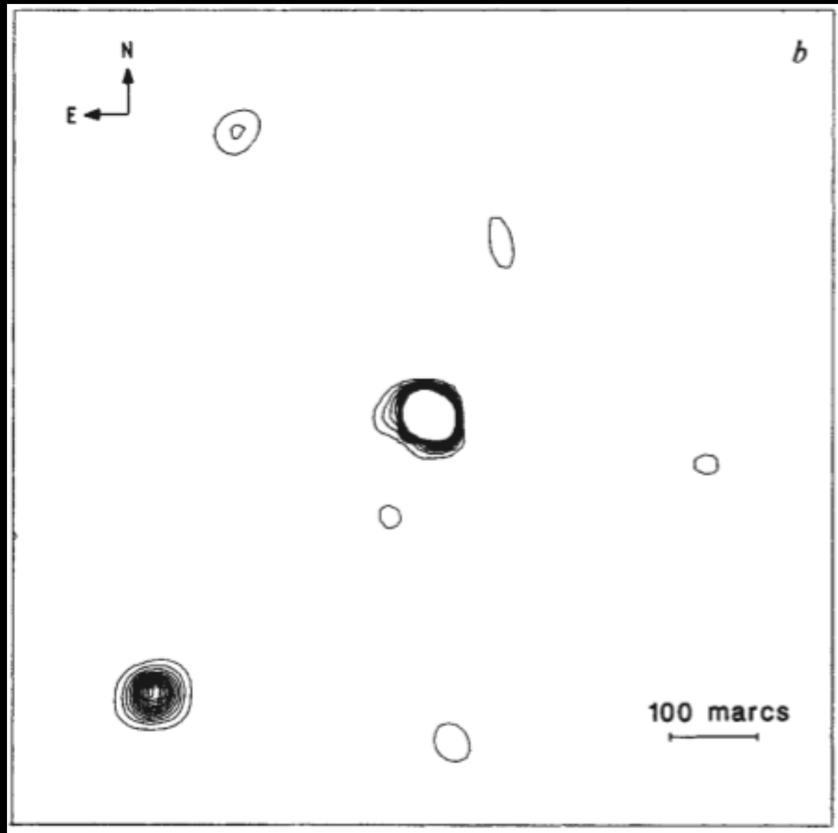
Motivation: Resolution is Key for Young Stars

- Very few young stars inside ~140pc:
- 10 AU = 70 mas at 140 pc. ~1 $M_\odot$ ~ R=12, K=9.
- Probing these key separations requires *the biggest telescope* and *the highest angular resolution, highest contrast technique.*
- Losing a factor of e.g. 2 in contrast for the sake of a neat algorithm does not apply here.

History 1: Binary Star Imaging

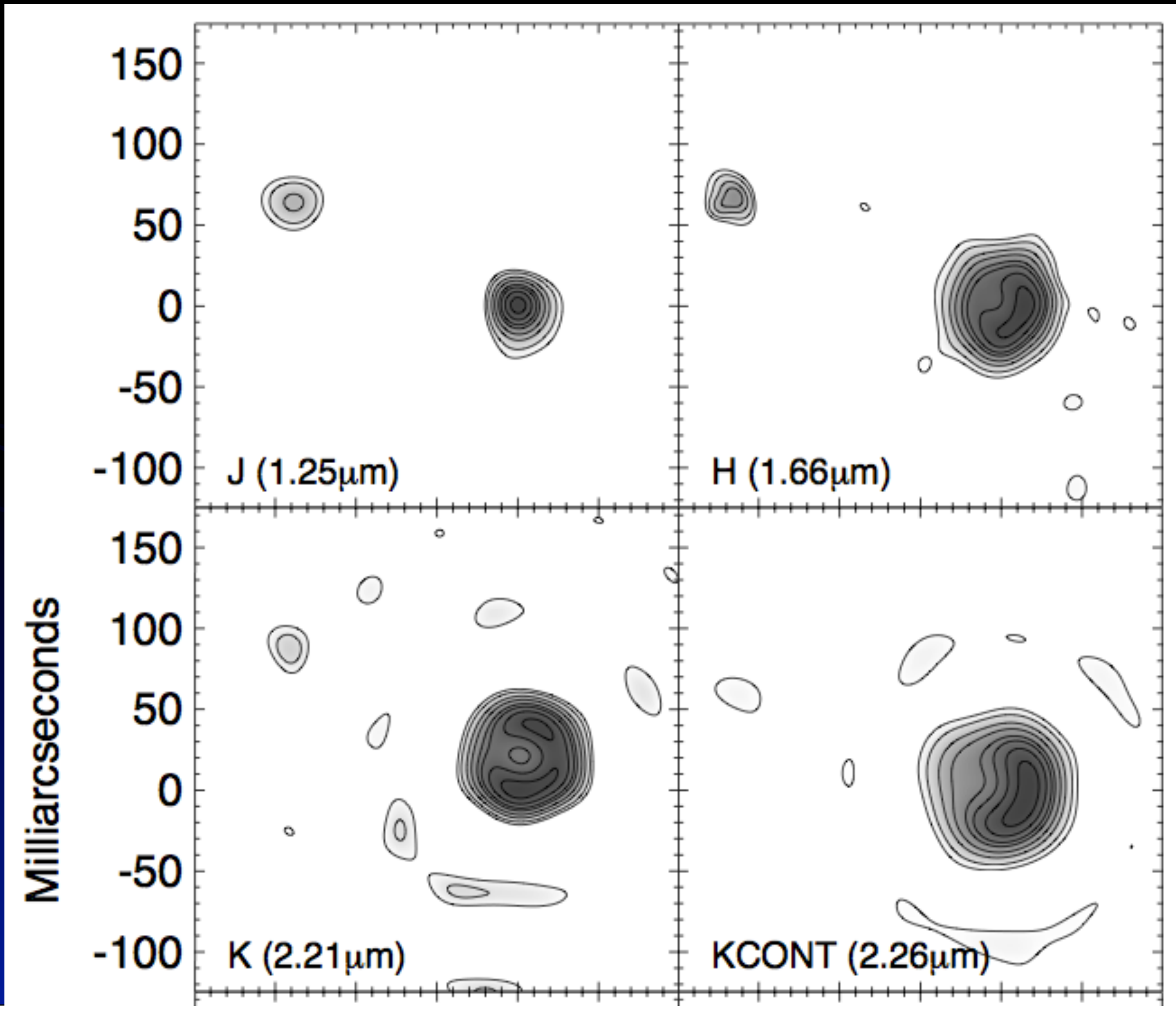
- Since the 1980s, radio interferometric techniques have been applied to SAM observations

Haniff et al (1987), ϕ And



History 2: Moderate Contrast Speckle Imaging

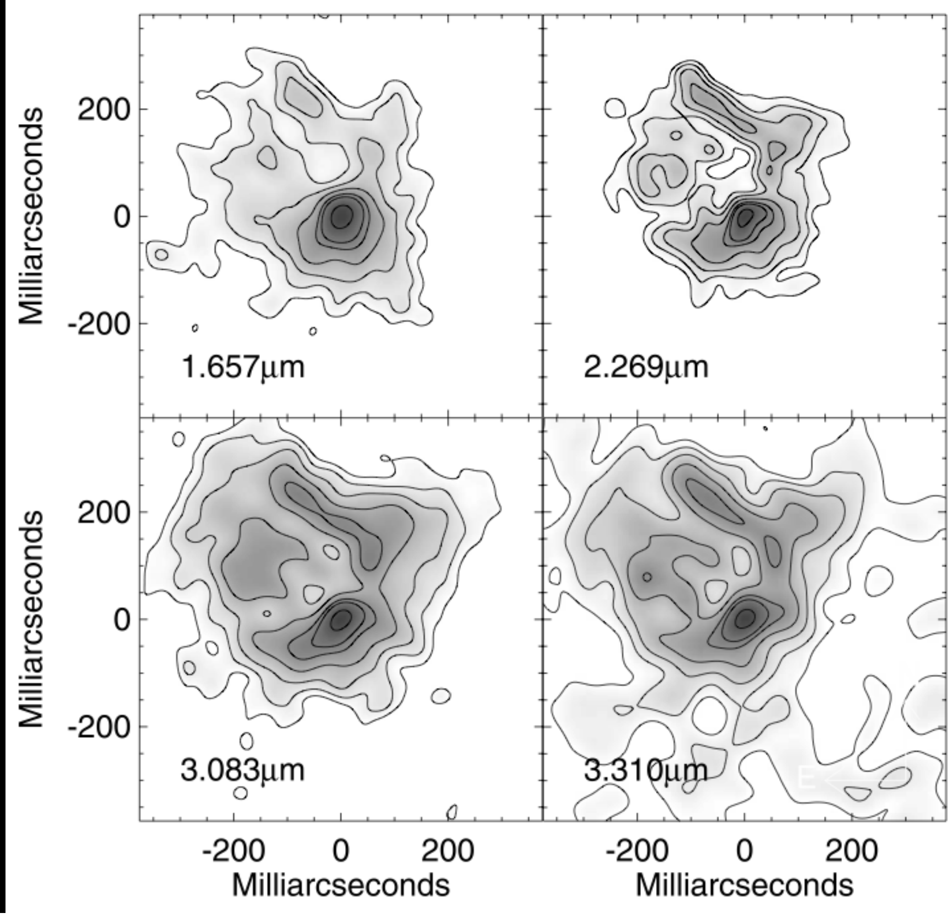
- LkHa 101 – a massive young star (Tuthill et al 2002)



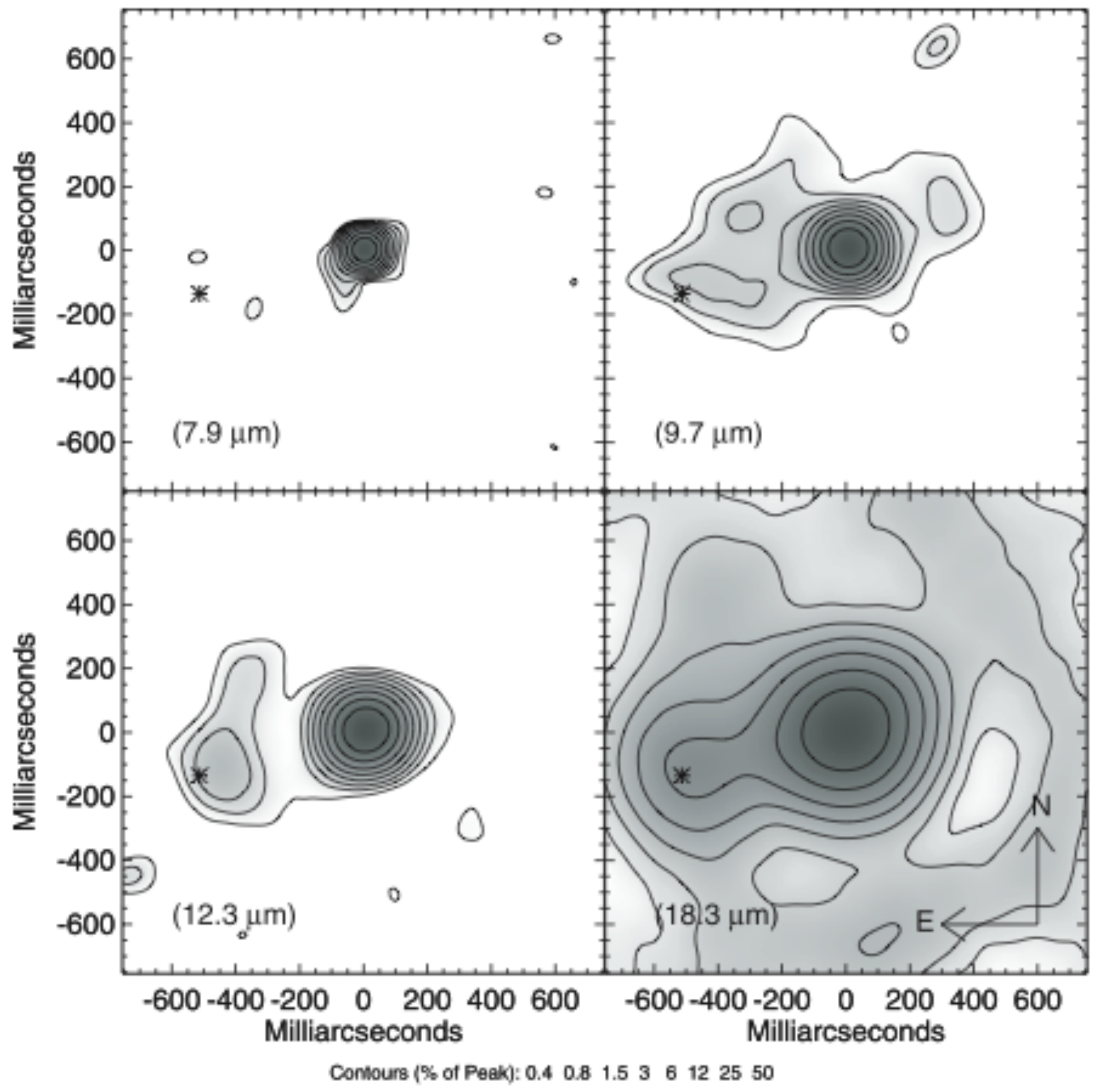
Lowest contour: 1%
Highest contour: 80%
1-sigma dynamic range ~100:1

History 2: Moderate Contrast Speckle Imaging

- MACIM was created originally to image IRC+10216, which sometimes broke VLBMEM.
- The difference was the *data* – less phase coverage meant that self-cal needed a good initial model.

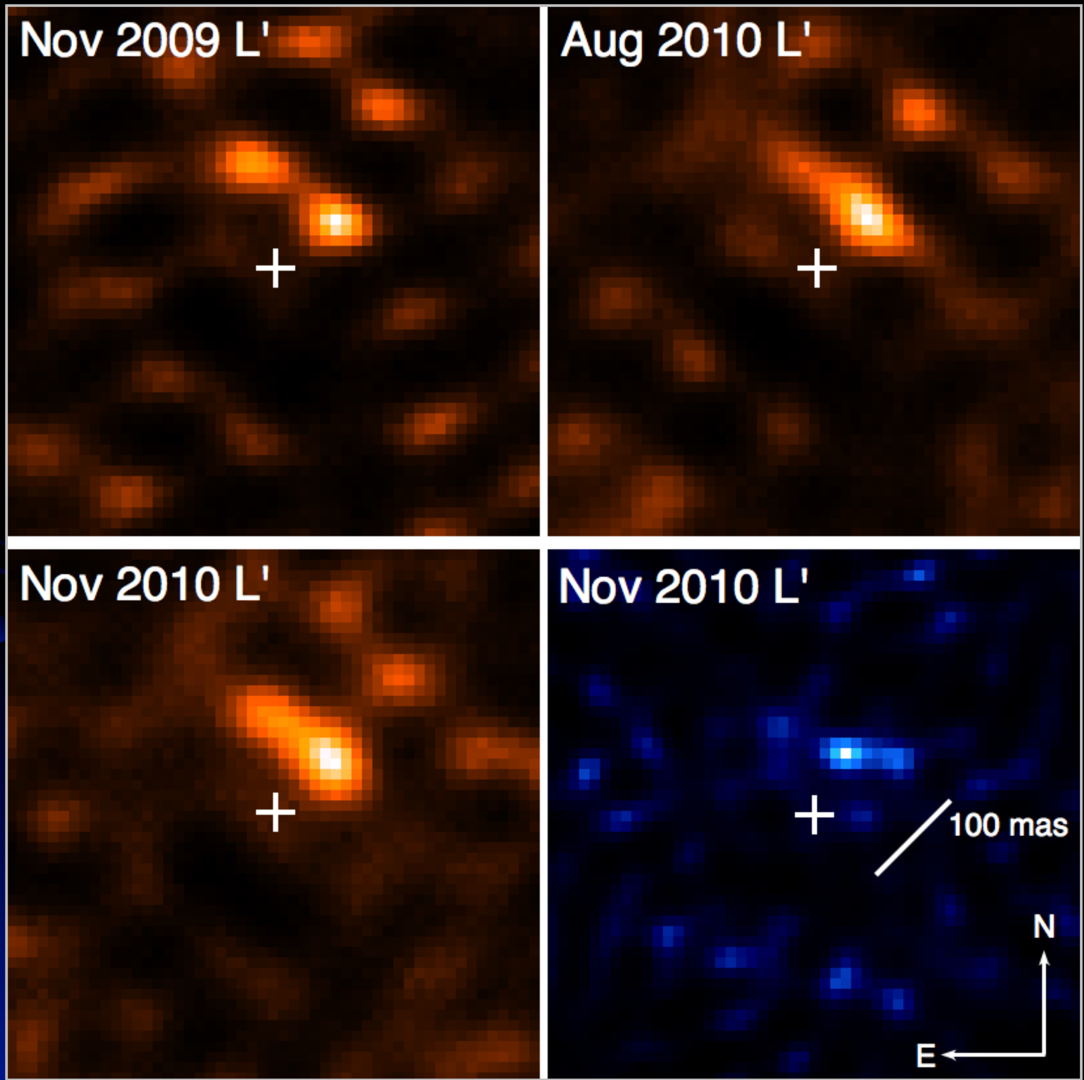


History 3: Hybrid Imaging



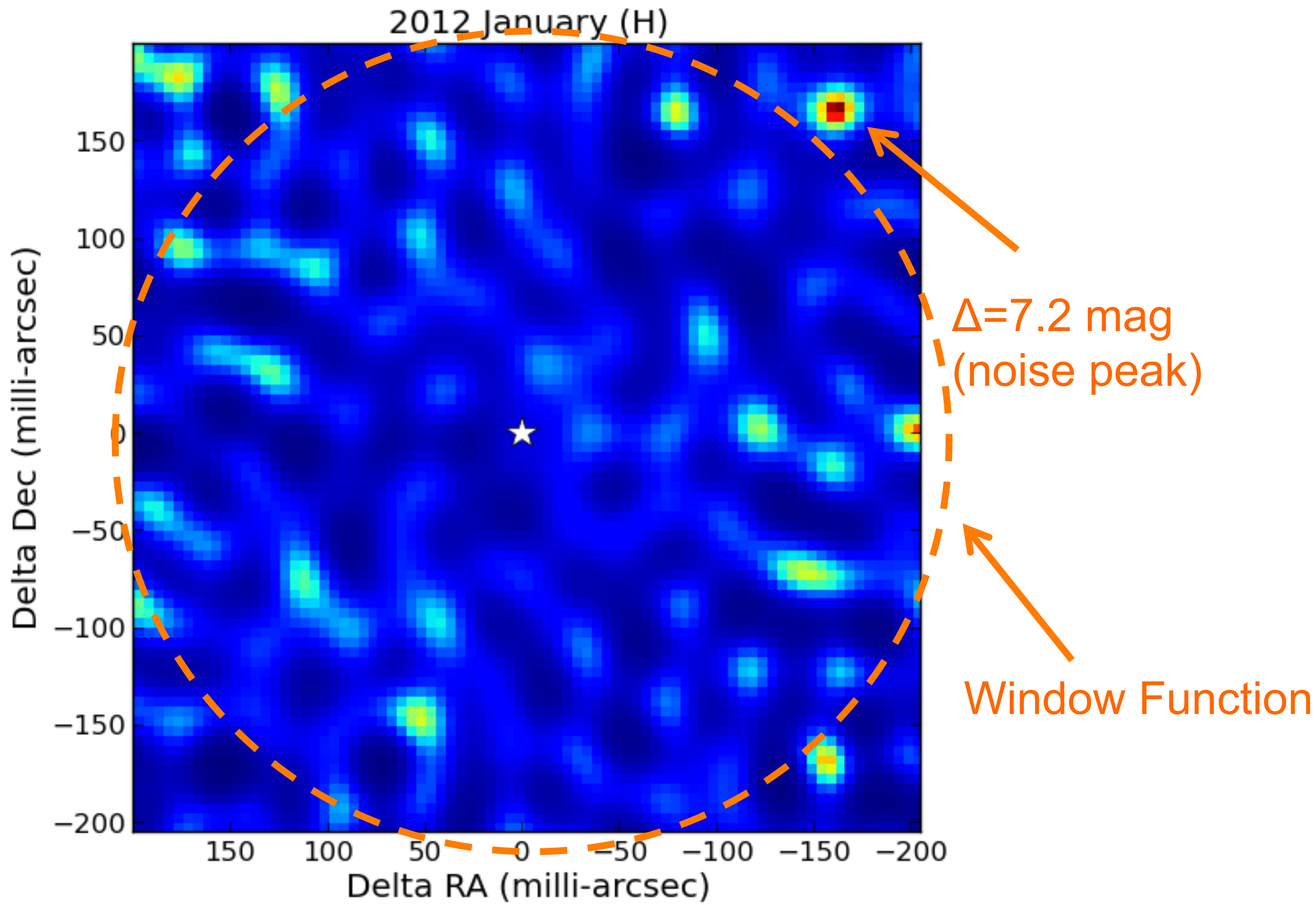
- For mid-IR work on Mira with Gemini/ Keck, imaging was much improved with a central uniform disk (or point-source).
- This was added back to the image.

History 3: Hybrid Imaging

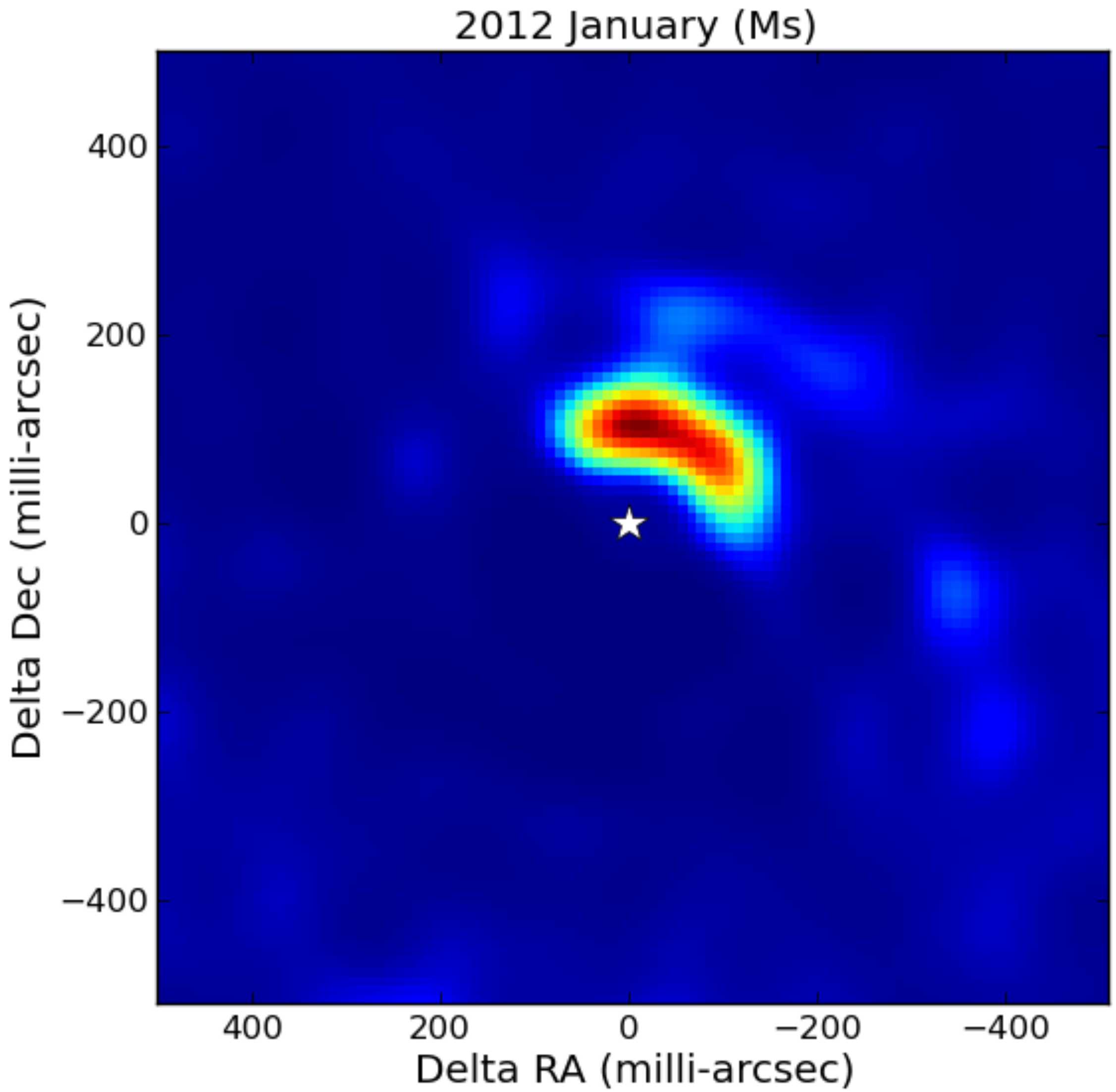


- For LkCa 15, we didn't bother showing the central point source, which would swamp the dynamic range.
- The Maximum Entropy regulariser penalises point sources – using any prior information in imaging is always powerful.

LkCa15 Update…
H-band (Jan 2012, nearly 6 *hours*)

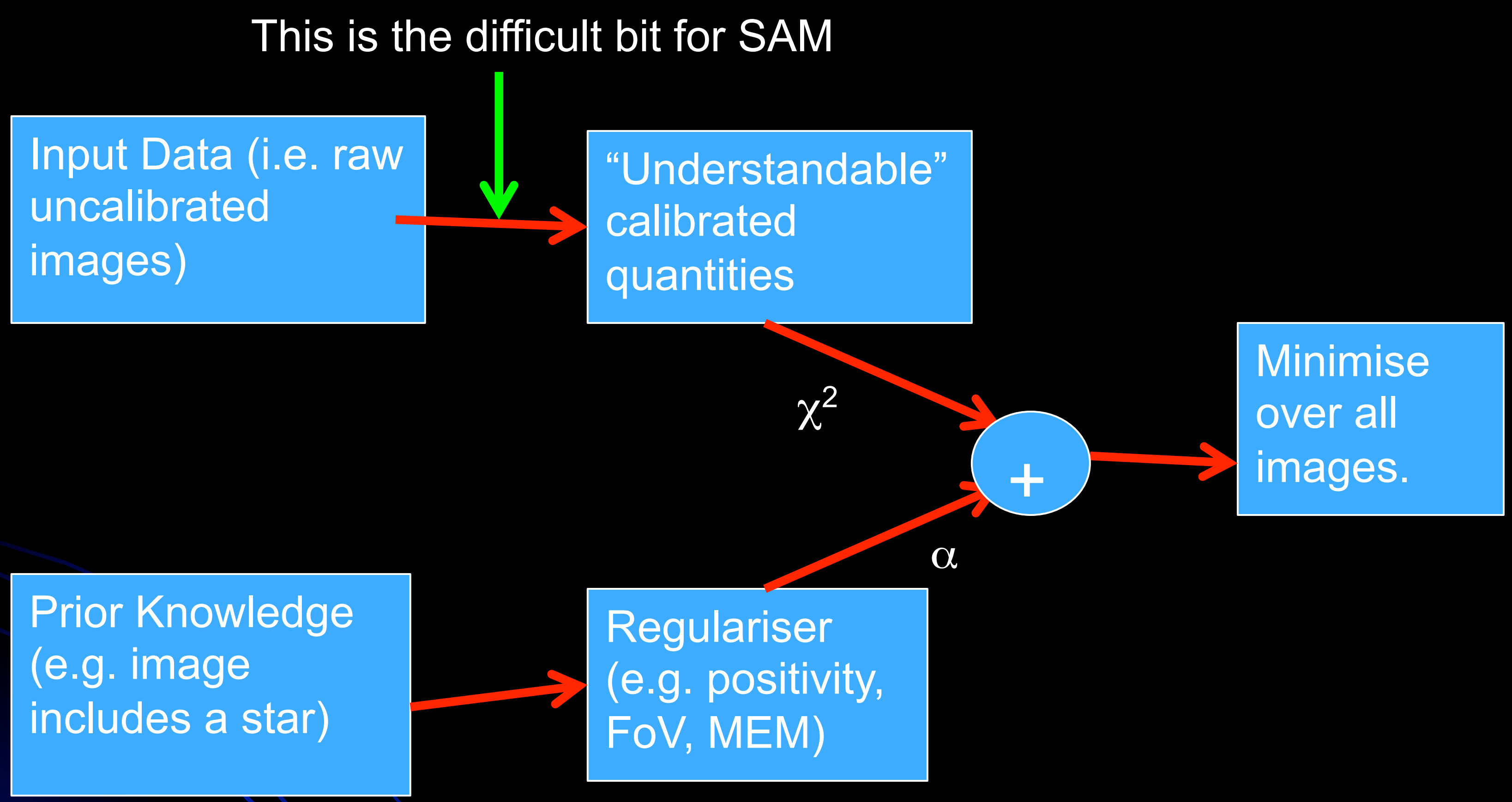


LkCa 15 Update…
M-band (Jan 2012)

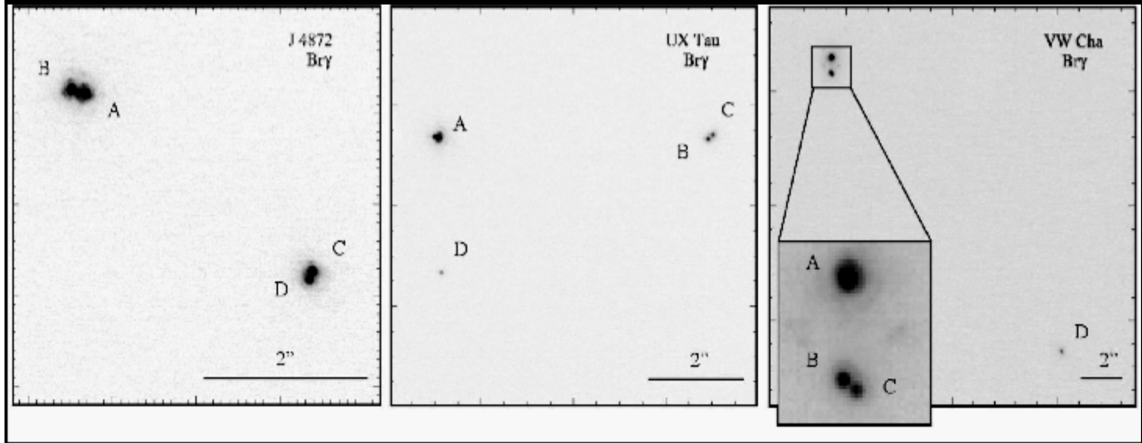


- Kernel-phase (full pupil) imaging
- Contrast Δ= 3.5 mag
- ~80 milli-arcsec arc similar to L-band

Imaging Principles

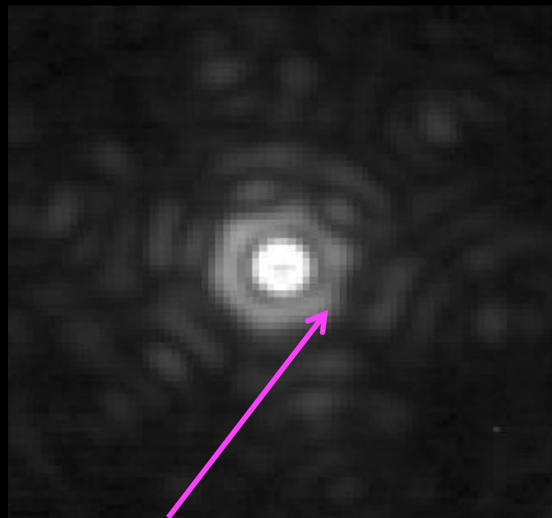


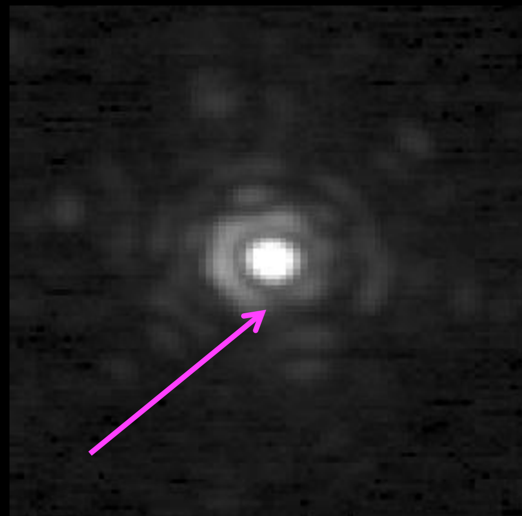
AO Imaging



2000s: AO imaging on 8-10m telescopes
(60 mas; 10 AU @150 pc)

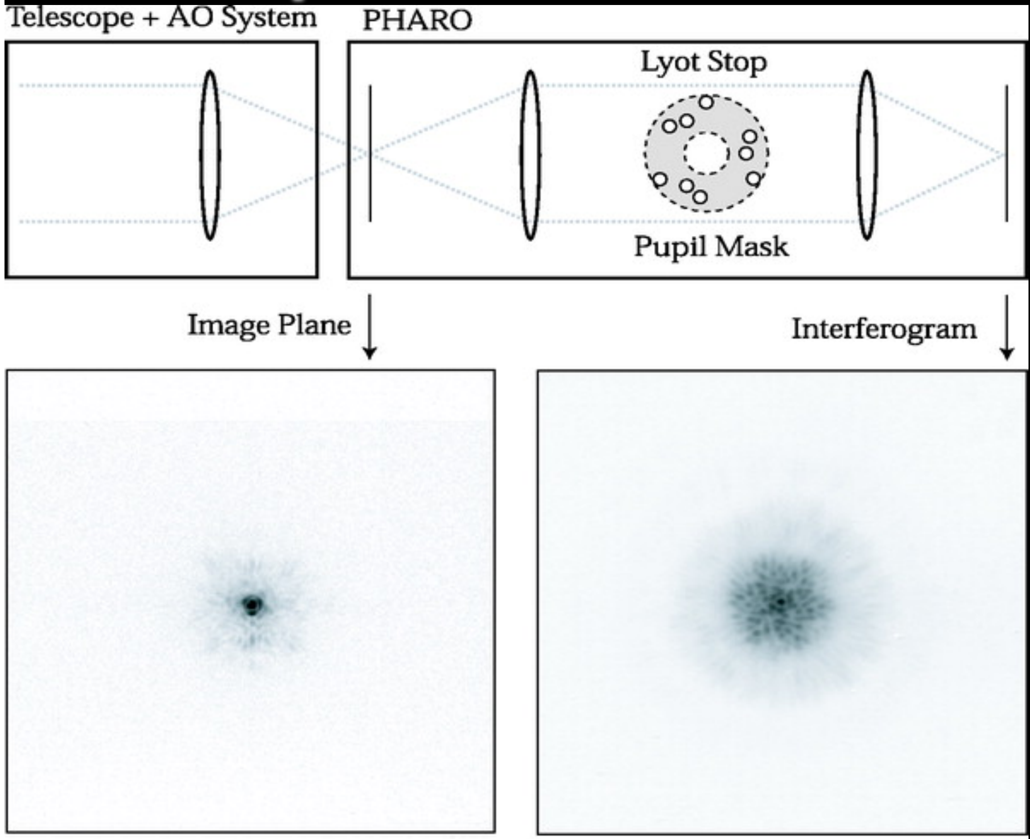
Correia et al. (2006)





AO images taken with same settings have subtle differences

Aperture Mask Interferometry

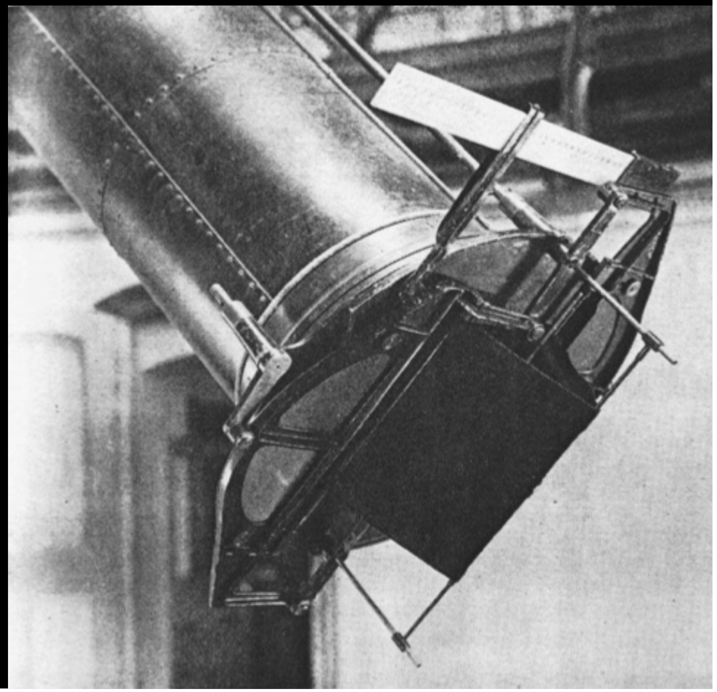


Placing an aperture mask in the pupil plane filters atmospheric turbulence on scales larger than the subapertures.

2010s (15 mas; 2 AU @150 pc)

Decomposing an interferogram into fringes for each aperture pair slit allows precise analysis, even if the wavefront piston over each sub-aperture is many radians.

Right: Michelson in ~1890…



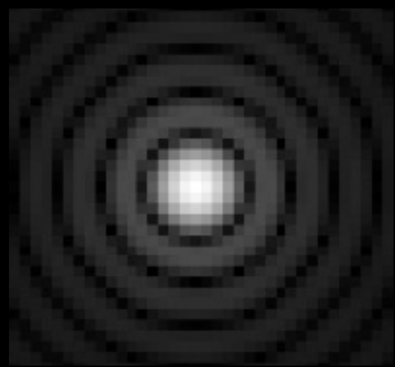
What about ADI etc?

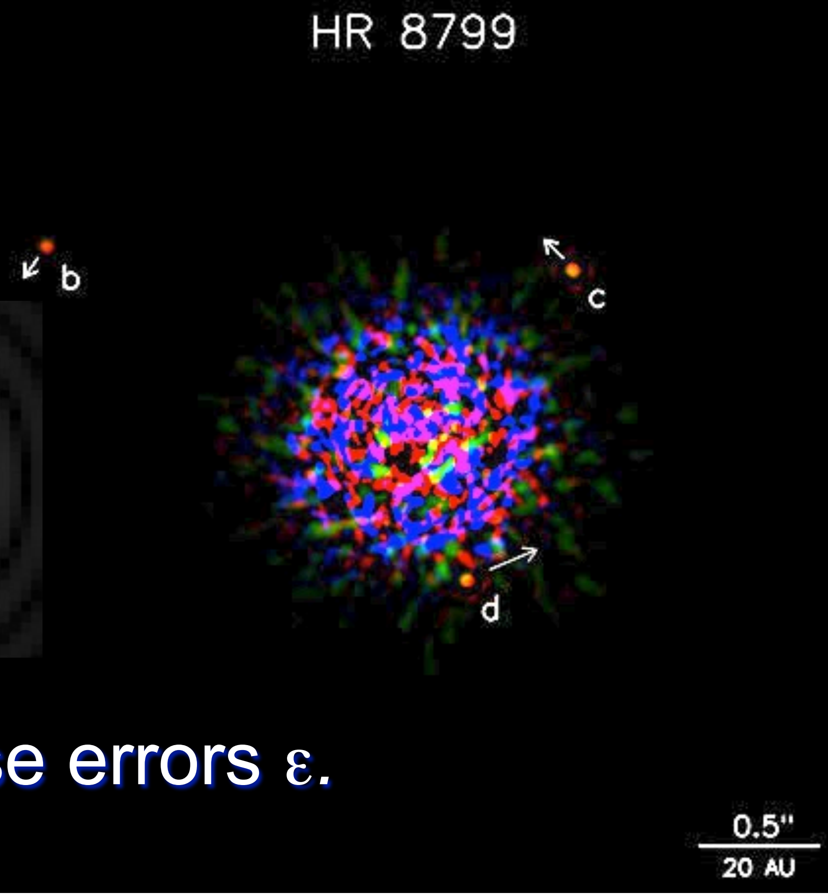
- At larger separations (~4 times the diffraction limit), Angular Differential Imaging and "LOCI" have been very successful.
- But a speckle at (x,y)

has amplitude ~

$|E_0(x.y)+ E_\sigma(x.y)|^2$, with

$E_\sigma = \mathcal{F}\,[\varepsilon(u,v)\,]$



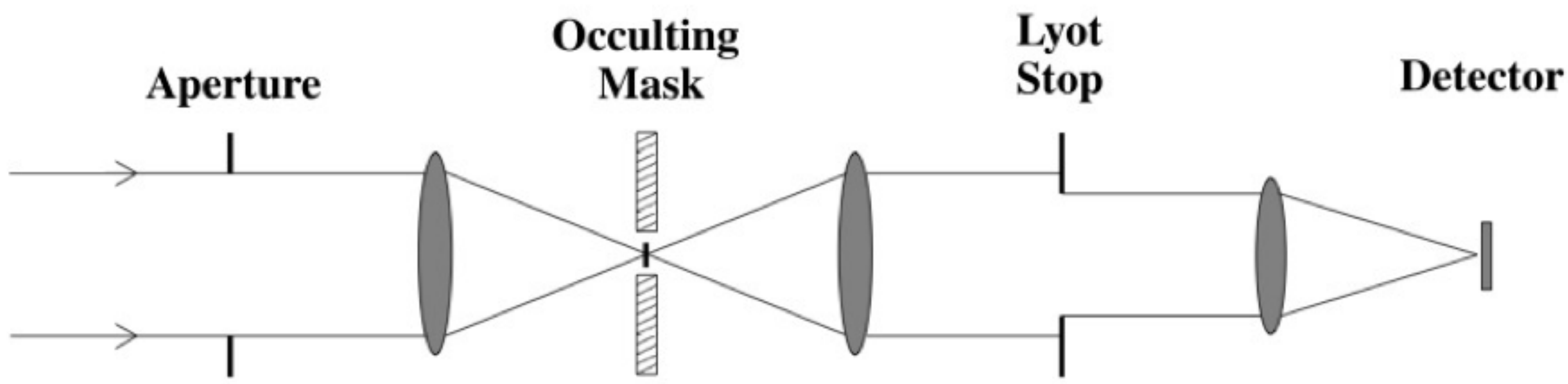


- Speckles are

first-order in pupil-plane phase errors ε.

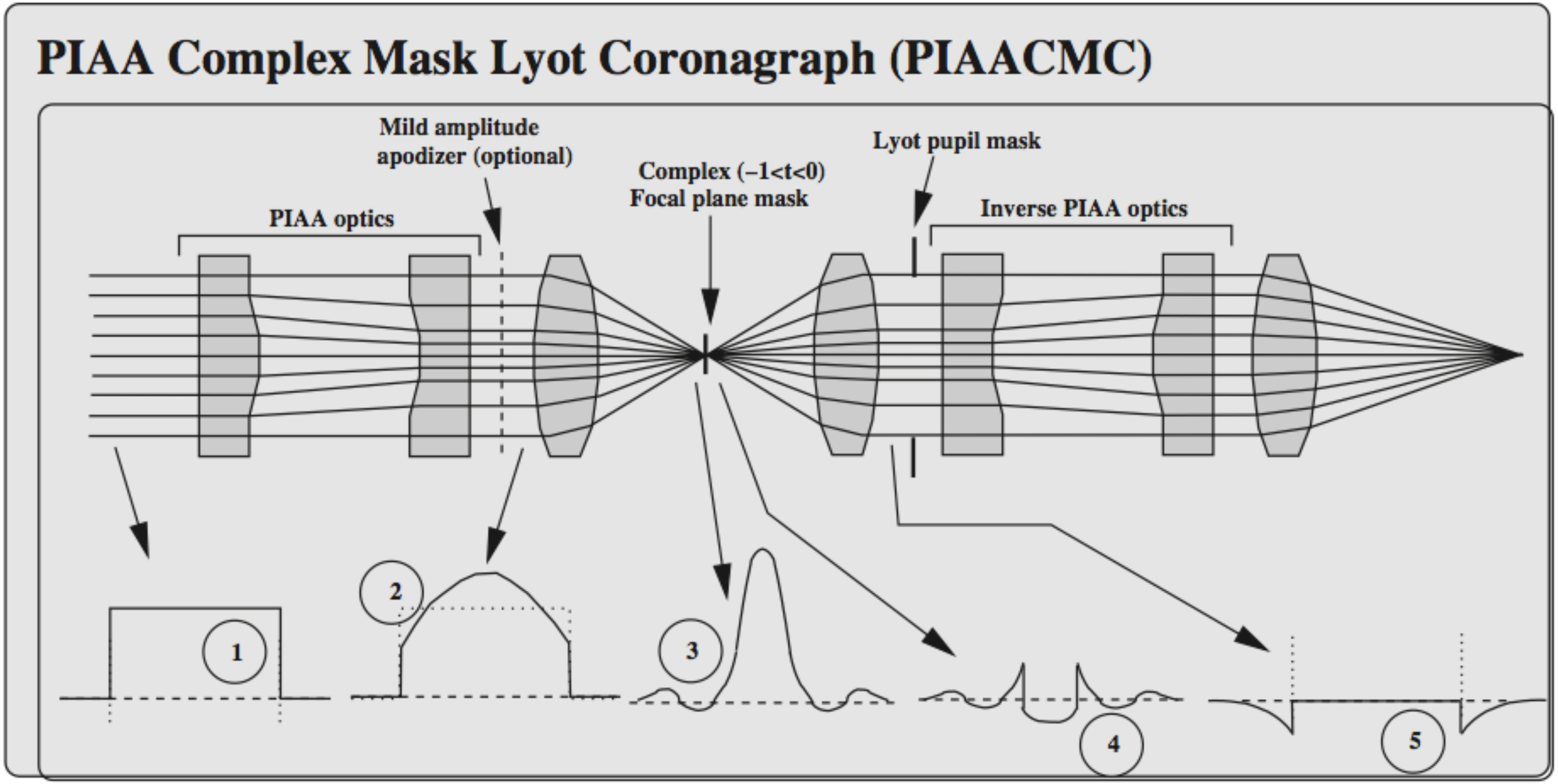
What about Coronagraphy?

- Coronagraphy eliminates the off-axis diffraction-limited electric-field E_0.
- Errors are *second-order* in pupil-plane phase-abberation ε… if the coronagraph is perfect.

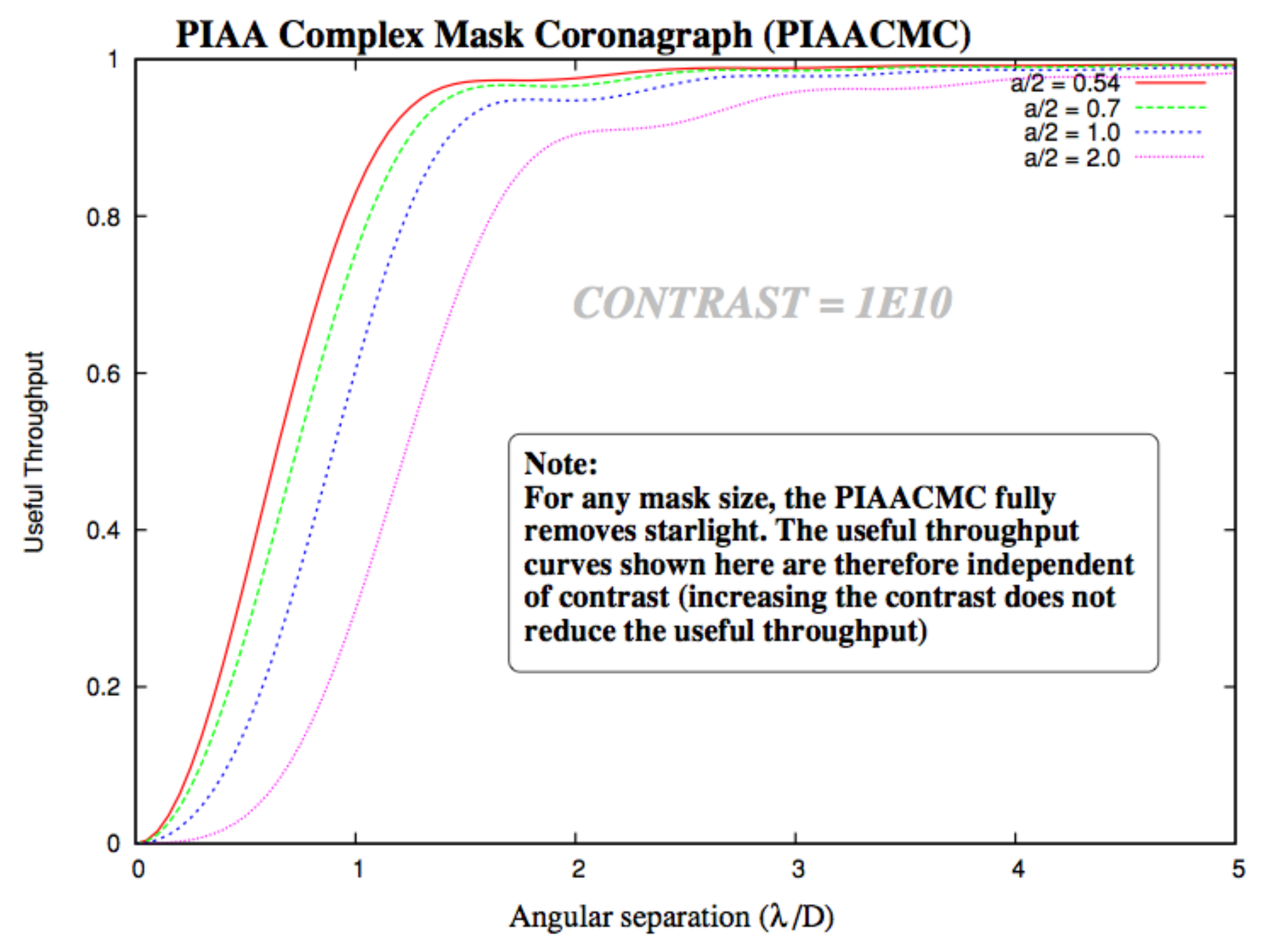


(From Sivaramakrishnan et al 2001)

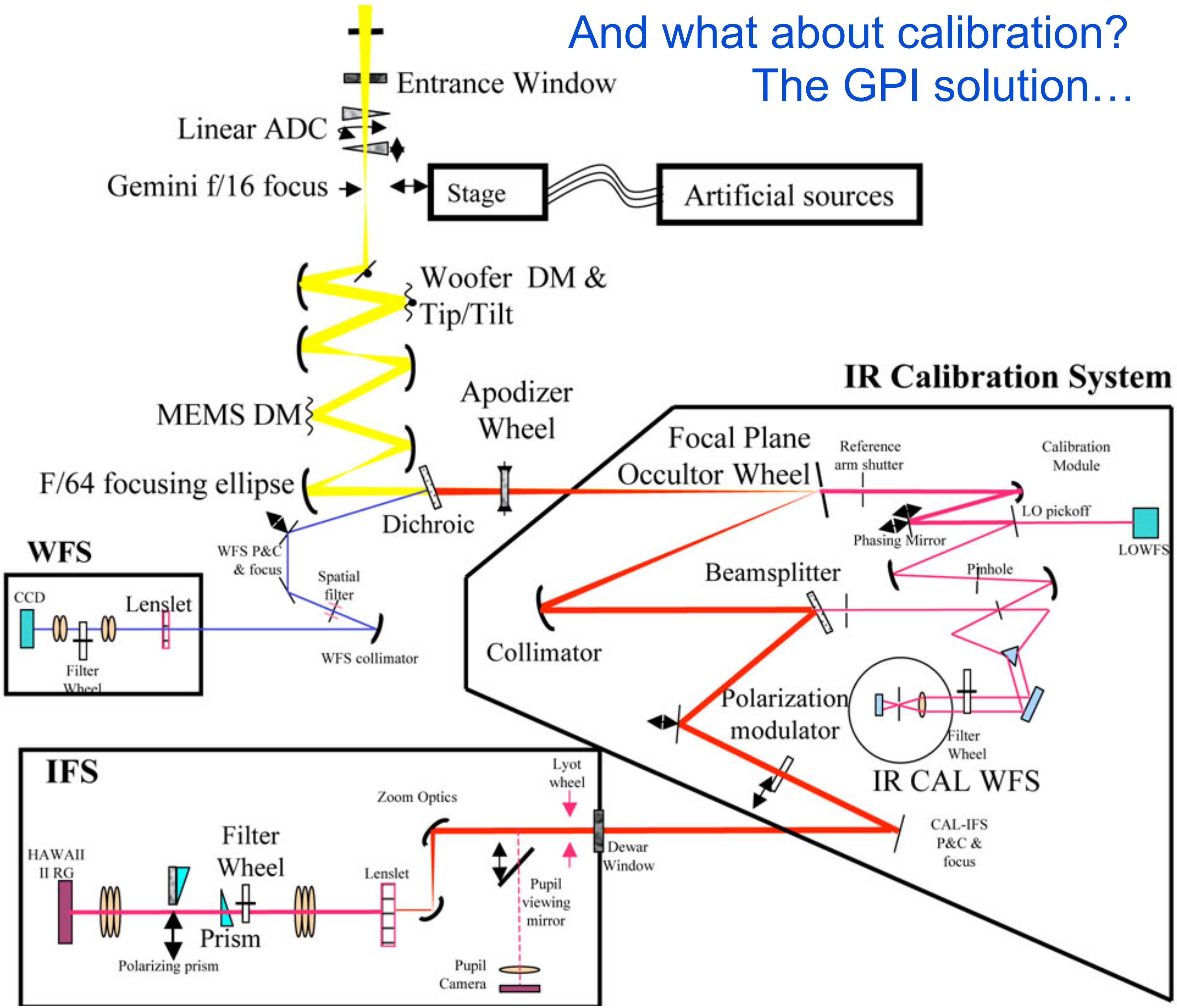
Perfect Coronagraphy…
(Guyon et al 2010)



Performance is perfect *IF no phase errors.*



And what about calibration? The GPI solution…

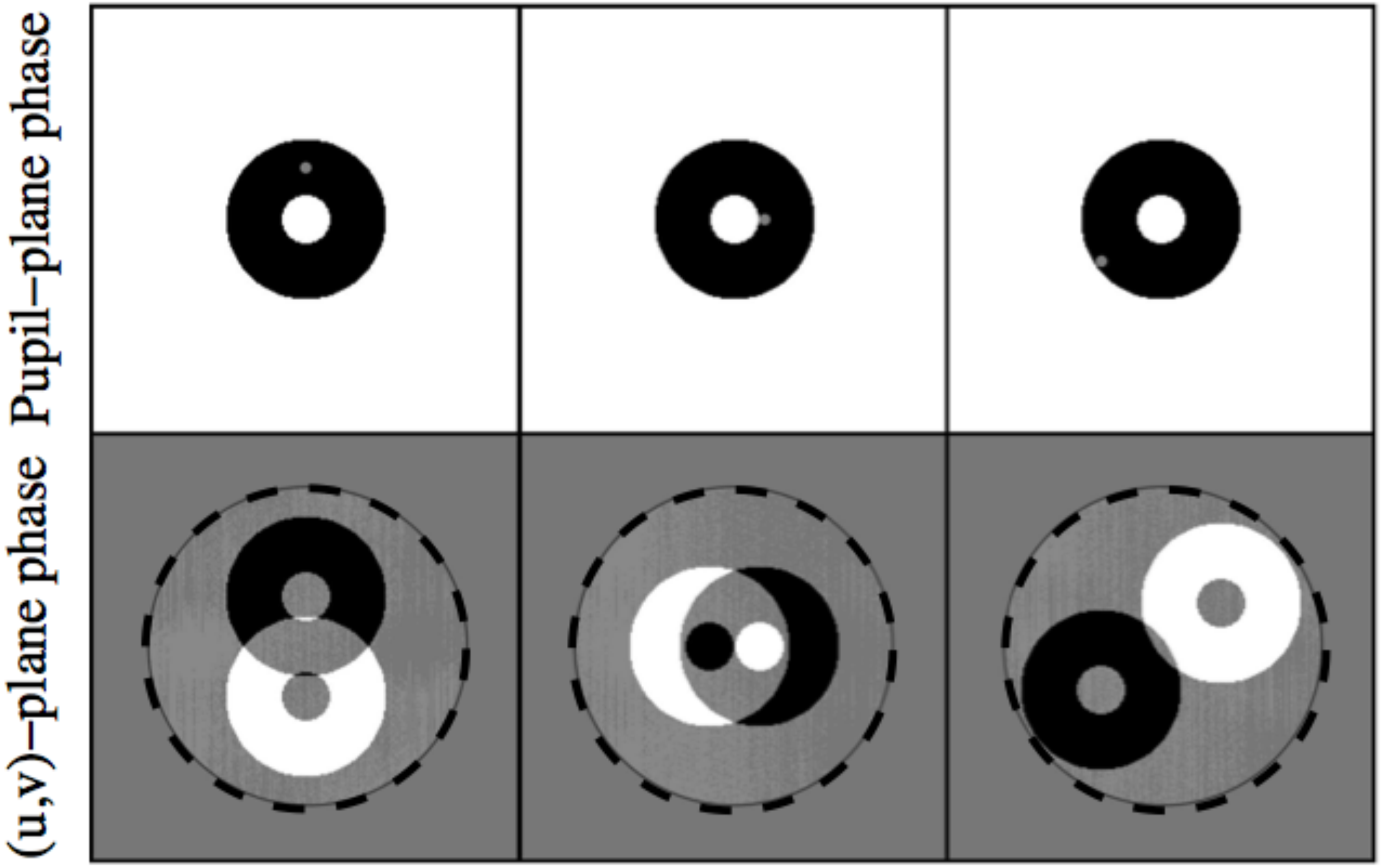


Kernel Phase

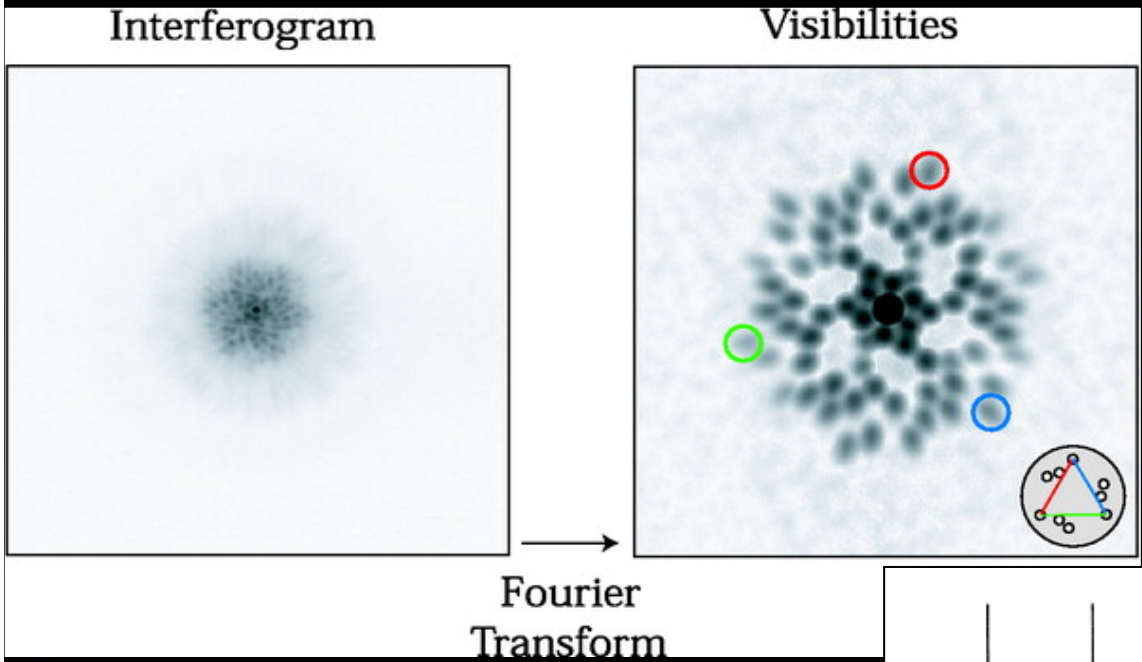
There is *always* more phase information in the (u,v)-plane than the pupil-plane. This extra information is the *Kernel-Phase* (Martinache 2010)

At least *half* of the Fourier phase info. is independent of pupil phase (1st order)

$$\mathbf{\Phi_F = A \cdot \Phi_P}$$



Closure Phases – a (correlated) example of Kernel-Phase

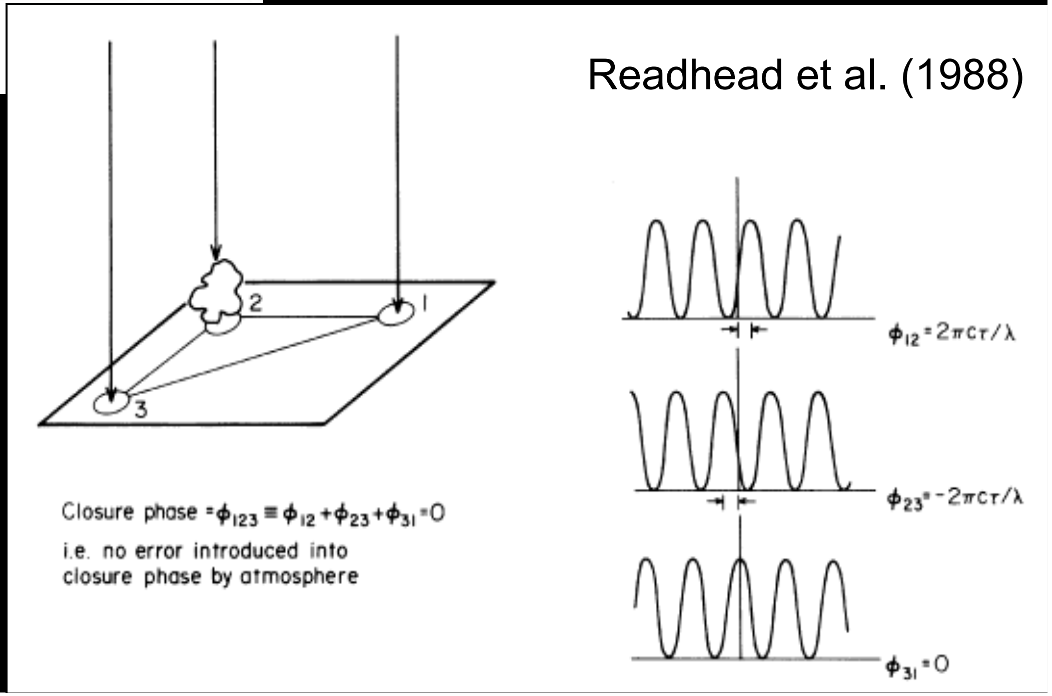


$$\Phi = \Phi_{12} + \Phi_{23} + \Phi_{31}$$

In the closure phase, any phase error on aperture 1, 2, or 3 will cancel out.

Almost all noise sources (esp. speckles) are just phase errors that readily cancel.

Readhead et al. (1988)



Limitations…

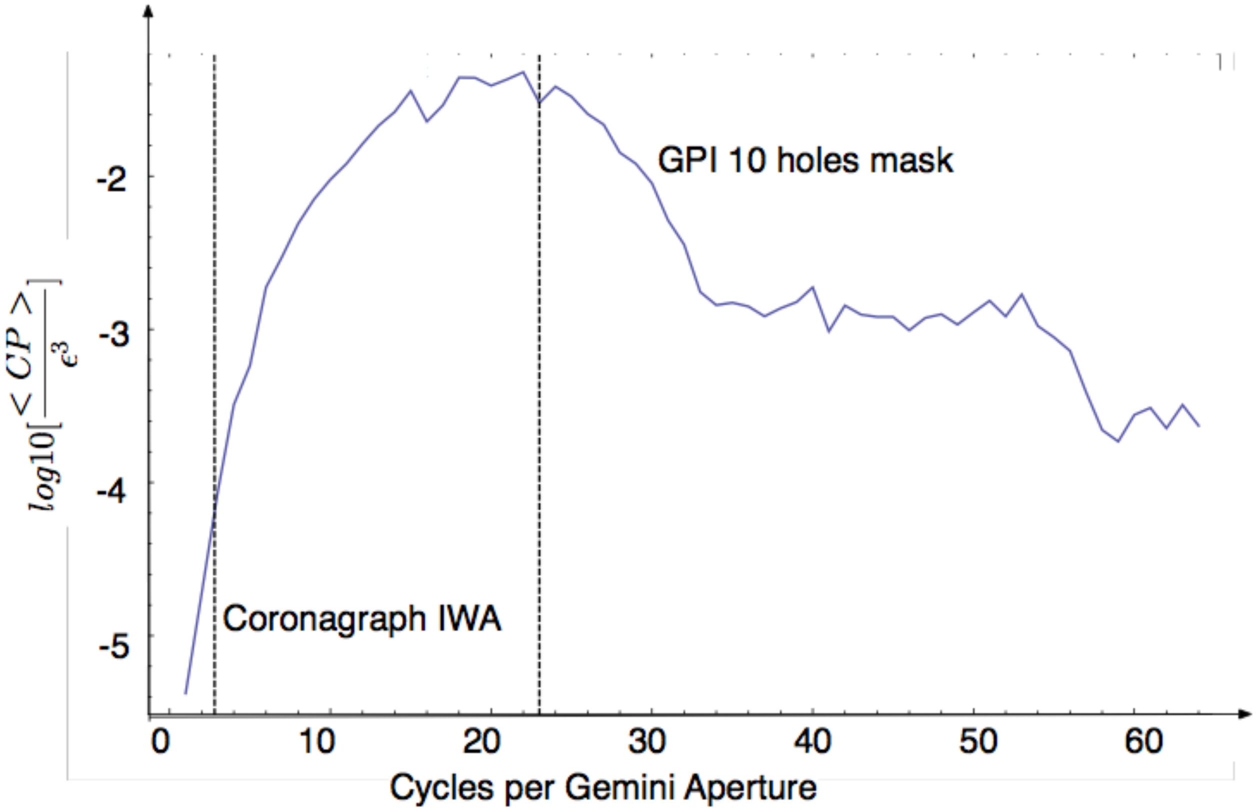
- Photon-noise… √N limit to contrast. N is small with a mostly opaque mask.
- Temporal phase errors:

$$\sigma(\phi_{\mathrm{cp}}) = \sqrt{3/GT}(1 - S_T)^{3/2}$$

- Variable spatial wavefront errors on the scale of a sub-aperture.

Third-order in ε.

Right: Sensitivity to quasi-static aberrations, expressed as closure-phase in radians divided by mean-cubed phase aberration, as a function of spatial-frequency.



Practical Correlations

- So far, this has been theoretical… in practice all visibilities and all kernel-phases are not created equal.
- Examples are:
  - Photon shot-noise, which means variance near the image center is higher.
  - A finite tip/tilt bandwidth (i.e. smaller than other AO modes).
- In the high Strehl regime, one could just take the real and imaginary Fourier components of the image, and use the data itself (i.e. of calibrators) to see which linear combinations have small uncertainties.
- But – splitting AO or speckle imaging data into pupil-phase, kernel-phase and visibility works for lower Strehls.

NB: Split for LkCa 15, K-band is ~0.5 radian phase, V2 error 0.06, closure-phase uncertainty 0.006

Practical Correlations

Ireland (2013): convert kernel-phase to statistically independent kernel-phase.

Projection matrix (e.g. eigenvectors of covariance matrix)

Uncorrelated observables for fitting/imaging

$$y_k = K_S \theta_p$$

Correlated observables

Practical Correlations

$$y_k = K_S \theta_p$$

$$s_i^2(\{y_k \forall k\}) > s_i^2(\{y_k : k \in C_j\})$$

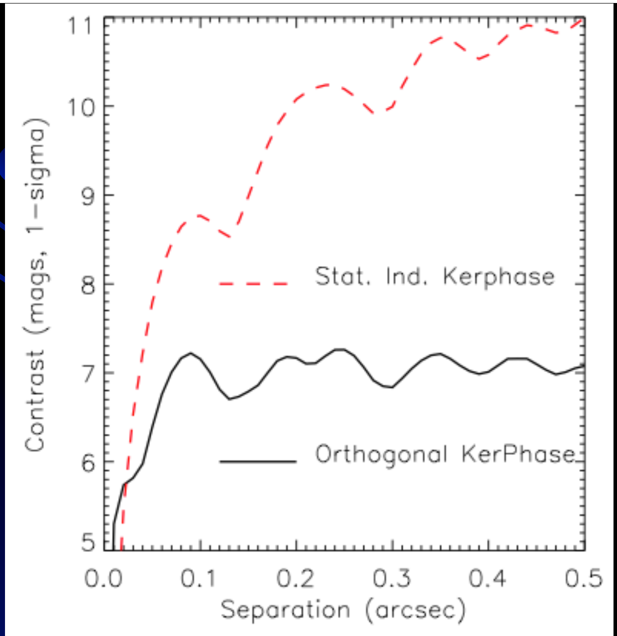
Ireland (2013): use the data itself to find poorly-measured and well-measured data types.

1. Ignore kernel-phases y_i whenever

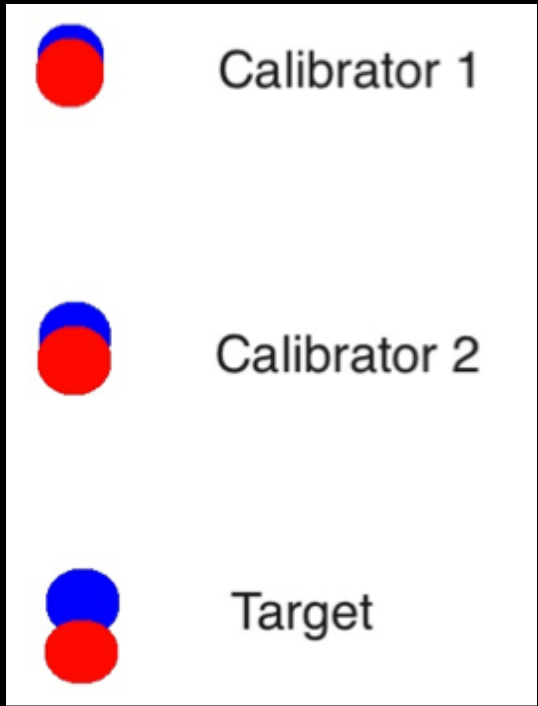
$$\delta_i^2 > \beta \langle s_i^2(\{y_k : k \in C_j\})\rangle_j. \quad (40)$$

   A typical value for β is 1, which rejects approximately 2 to 3 out of 28 kernel-phases for 9-hole Keck aperture-masking data.

2. Add δ_i^2 to each target observation's uncertainty estimate for the remaining kernel-phases i.

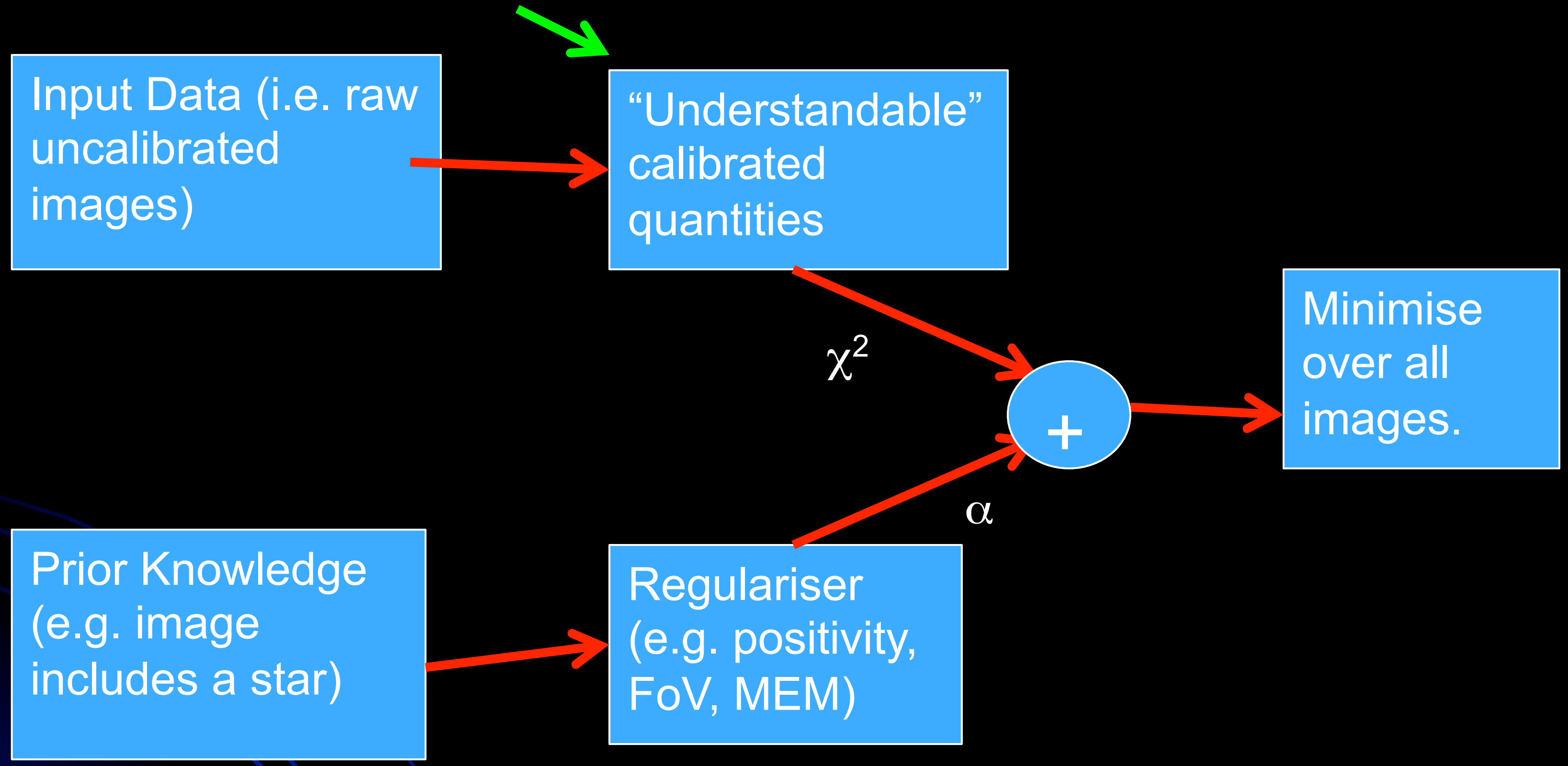


Karhunen-Loeve eigenimages/ POISE



Imaging Principles

Statistically independent kernel-phases and V^2 with its covariance



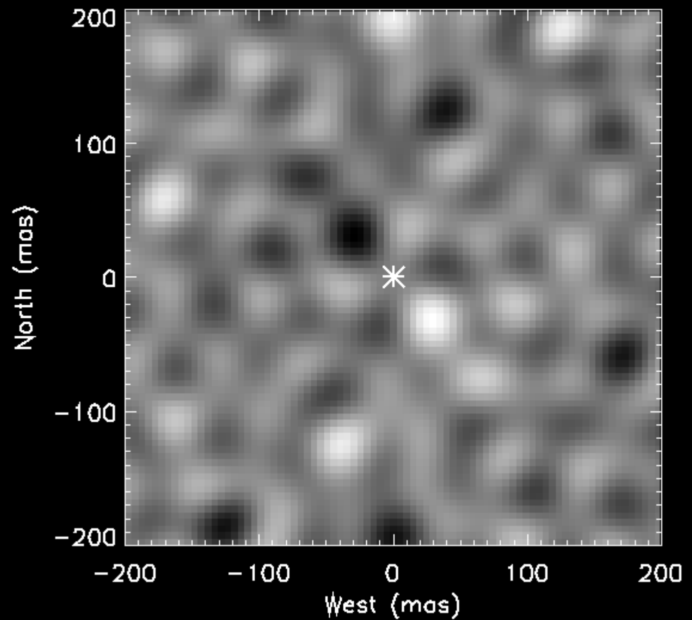
Symmetrical/Asymmetrical Imaging

- In the high-contrast regime:
  - Amplitude-> point-symmetric image component
  - Phase->point-antisymmetric image component.
- This is great for experts, but how to publish?

Below: Map for one independent kerphase
Peak contrast = 8 radians/unit contrast

$$I_s = (I_0 + I_{180})/2$$
$$I_a = (I_0 - I_{180})/2.$$

$$|V|(\mathbf{u}) \approx 1 + \int I_f(\mathbf{x}) \cos(2\pi \mathbf{u}\cdot\mathbf{x}) d\mathbf{x}/F_s$$
$$\phi(\mathbf{u}) \approx \int I_f(\mathbf{x}) \sin(2\pi \mathbf{u}\cdot\mathbf{x}) d\mathbf{x}/F_s$$



Conclusions

- There is nothing special about imaging from SAM when compared to other interferometry data: all imaging is hard.
- Correlations between data products are more obvious and important for SAM.
- The challenges are:
  1. Finding the right data products that include all covariances.
  2. Fast algorithms that don’t take too much code maintenance…