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Diophantine optics in nulling interferometry: laboratory performances of the chessboard phase shifter

D. Rouan[1], D. Pickel[1], D. Pelat[2], J.M. Reess[1], F. Chemla[3], M. Cohen[3] & O. Dupuis[1]

[1]*LESIA - CNRS - Observatoire de Paris - UPMC - U. Paris-Diderot, Meudon, France*

[2]*LUTH - CNRS - Observatoire de Paris - U. Paris-Diderot, Meudon, France*

[3]*GEPI - CNRS - Observatoire de Paris - U. Paris-Diderot, Meudon, France*

Abstract. Among the techniques of very high angular resolution to isolate planetary photons from those of the star - several million times more numerous - a possible one is the nulling interferometry with several telescopes in space. This mode requires an achromatic phase shift of pi in one arm of the interferometer. We introduced several years ago the concept of quasi-achromatic phase shifter based on two chessboard mirrors, each cell introducing a phase shift determined by a mathematical law, so that the behavior of the attenuation of the star as a function of wavelength is flat in a range of more than one octave. This concept is part of broader field that we call diophantine optics and that will be first presented. We then present the experimental validation of such a concept where the chessboard phaser is synthesized using a properly controlled deformable segmented mirror from Boston micro machine with 12x12 actuators. First we introduce the principle of the "chessboard effect", then the dedicated testbench DAMNED will be described in particular the control of the phase through strioscopy. The results on the performances of the device will be presented and the ways of improvement and extension of this concept will be analyzed.

1. Introduction

To find one day clues of life on extra-solar planets, or at least characterize them, we need in the future to directly detect photons from exoplanets so as to obtain spectra where specific spectroscopic biomarkers could be found. Among the techniques of very high angular resolution to isolate planetary photons from those of the star - millions to billions times more numerous - a possible one is the nulling interferometry with several telescopes in space. The principle: if the star coincides with a dark fringe it is strongly attenuated, while the planet on a bright fringe is not. This mode requires an achromatic phase shift of π in one arm of the interferometer. We introduced in 2008 the concept of a quasi-achromatic phase shifter based on two chessboard mirrors, each cell introducing a phase shift determined by a mathematical law, so that the behavior of the attenuation of the star as a function of wavelength is flat in a range of more than one octave. This concept is part of broader field that we call diophantine optics and which is first presented.

2. Diophantine optics

Diophantus of Alexandria is a greek mathematician, known as the father of algebra. He studied polynomial equations with integer coefficients and integer solutions such as $(x-1)(x-2) = x^2 - 3x + 2 = 0$, called *diophantine equations.* The most famous one is the aegyptian triangle $5^2 = 4^2 + 3^2$ which allowed the builders of the pyramids to make perfectly squared monuments. What could be the relationship between optics and power of integers ? In fact, in optics, constructive or destructive interferences imply that optical path differences (*opd* in the following) are multiple integer (odd or even) of $\lambda/2$ and, besides that, the complex amplitude is a highly non-linear function of the *opd* so that any Taylor development implies powers of integers. This is where diophantine equations appear. The definition one can give of diophantine optics is *the exploitation in optics of some remarkable algebraic relations between powers of integers.* It happens that its application to techniques aiming at direct detection of exoplanets are numerous, probably because of a bias of the first author towards the field of exoplanetary sciences. The website *http://dan.rouan.free.fr/OptiqueDiophantienne/intro.html* gives (in french) a good idea of the basics and some applications, as well as litterature. In the following we give several examples where the concept of diophantine optics has been used.

2.1 The hard task of detecting an exoplanet directly

Because of both the extremely high contrast between the planet and the star (10^{10} in visible and 10^7 in thermal infrared) and the angular proximity of the two objects (sub-seeing), it is well established that direct detection is an extremely difficult problem. Since the mid of the 90's, two main avenues were considered to solve it: in visible/near-IR, the use of single telescope plus adaptive optics and coronagraphy (e.g. Guyon 2007) and in the thermal infrared, the use of coherent recombination of several telescopes in space in a nulling interferometric mode (Bracewell 1978; Leger & Herbst 2007; Cockell et al. 2009). Let's have a look to the kind of plus that diophantine optics can bring to each of those two techniques.

2.2 Achromatization of a Four-Quadrant Phase Mask Coronagraph

The Four-Quadrant Phase Mask Coronagraph (FQPMC) was introduced by one of us in 2000 (Rouan et al. 2000). In a certain sense it is already an application of diophantine optics principles since it allows to cancel out to the first order the light of an on-axis star by producing destructive interferences between four sub-beams. This is done thanks to a special component, the four-quadrant phase mask, which is made, in its basic version, of two kind of plates, one for each diagonal, with between them an *opd* difference equal to $\lambda_o/2$; steps follows in the order [0, 1, 0, 1]. A classical FQPMC is by nature chromatic: when λ is different from λ_o the residual intensity is proportional to $(\delta\lambda/\lambda_o)^2$. Now let's play with combinations of quadrant thicknesses (see Fig. 1) to cancel the first

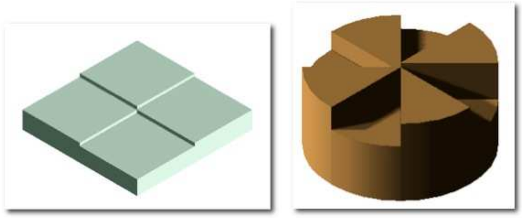

Figure 1.: *Achromatization of a 4QPMC and of a 8QPMC.*

terms of the Taylor development of the complex amplitude $a = \sum \pm \exp(jk\phi)$ where $\phi = \pi\delta\lambda/\lambda_0$ (Rouan et al. 2007). At second order, with steps [0, 1, 2, 1] we obtain $I = (\delta\lambda/\lambda_o)^4$ and at third order, a 8QPMC with steps [1, 8, 3, 6, 2, 7, 2, 7] gives $I = (\delta\lambda/\lambda_o)^6$. We have gained a lot in achromatism performances.

2.3 Avoiding long delay lines

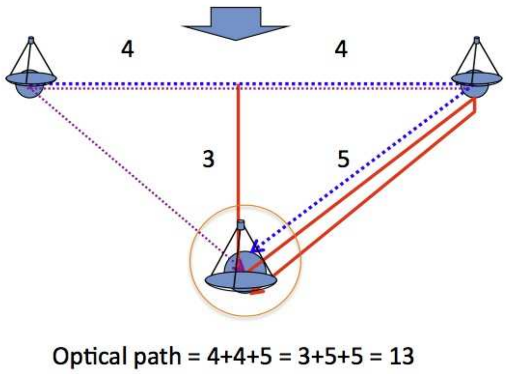

Figure 2.: *Avoiding long delay lines in a 3-telescopes nulling interferometer.*

In a 3-telescopes nulling interferometer in space, as the one proposed for the Pegase experiment (Ollivier et al. 2006), avoiding long delay lines or a fourth spacecraft for recombination is essential, and we proposed a genuine solution based on the Aegyptian triangle, as illustrated on figure 2. The idea is to install on the three spacecrafts, each carrying a telescope, mirrors that allow the light to “circulate” , so as to equalize the *opd* between the different beams before recombination on one of the spacecraft.

2.4 Deep nulling interferometers

In a Bracewell nulling interferometer, the stellar disk is generally resolved because of the long baseline. This leads to an uncomplete nulling of the stellar light because of leaks coming from the outer ring of the stellar disk. This is a well known problem and several configurations using more than two telescopes were

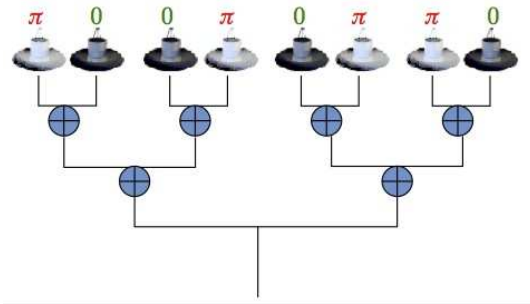

Figure 3.: *Deep nulling interferometers based on the Prouhet-Thué-Morse sequence*

proposed, such as the Angel's cross or the Mariotti's configuration which provide nulling angular function in θ^n with $n = 4$, instead of $n = 2$ in the Bracewell's configuration. There is indeed a universal rule to obtain any value of n in a multi-telescopes interferometer (Rouan 2007, 2004). It consists in distributing regularly 2^L telescopes on a straight line and to introduce a π phase shift to those which correspond to a "0" in the Prouhet-Thu-Morse sequence $0110100110010110\cdots$, as illustrated on Fig. 3. This sequence can be constructed recursively and has a fractal structure (Allouche & Shallit 1999). Nulling varies then as θ^n with n as high as wished. This is so thanks to the remarkable diophantine relation established by Prouhet (1851): $\sum m_0^p = \sum m_1^p$ where m_0 are all the ranks of "0" in the PTM sequence from 1 to 2^L and m_1 are all the ranks of "1". For instance $1+4+6+7 = 2+3+5+8$ and $1^2+4^2+6^2+7^2 = 2^2+3^2+5^2+8^2$. One then shows that the first coefficients, up to L-1, of the Taylor development of the complex amplitude vanish: $a = j\phi \sum (m_0 - m_1) - \phi^2 \sum (m_0^2 - m_1^2) - j\phi^3 \sum (m_0^3 - m_1^3) + ...$, so that the first non-vanishing term varies as $\phi^{L-1} \propto \theta^{L-1}$ when there are 2^L telescopes.

3. The achromatic chessboard

In nulling inteferometry, as introduced first by (Bracewell 1978), one key element is the sub-system that provides an *achromatic* π phase-shift in one arm of the interferometer. Several solutions were proposed, most of them implying an asymmetry between the two arms of the interferometer. A few years ago, we introduced a new concept of achromatic phase shifter, the achromatic chessboard (Rouan & Pelat 2008; Pelat et al. 2010; Pickel et al. 2013). It is based on a single optical device and some unforeseen application of diophantine equations. The wavefront is divided into many sub-pupils thanks to two "chessboards" of phase-shifting cells, each producing an *opd* that is an even (in one arm) or odd (in the other arm) multiple of $\lambda_o/2$, where λ_o is a central wavelength in the spectral domain where achromatic phase shift is desired. The main assets of this solution are a) to make the arms of the interferometer fully symmetric and b) to use a unique simple component that can be in bulk optics or consists in a single deformable mirror.

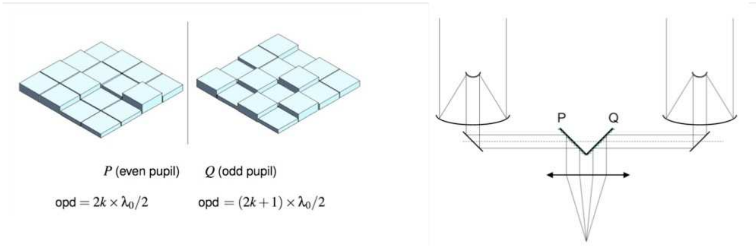

Figure 4.: *The chessboard achromatic phase-shifter; left: the pair of chessboard phase-shifters; right: its implementation in a Bracewell's interferometer.*

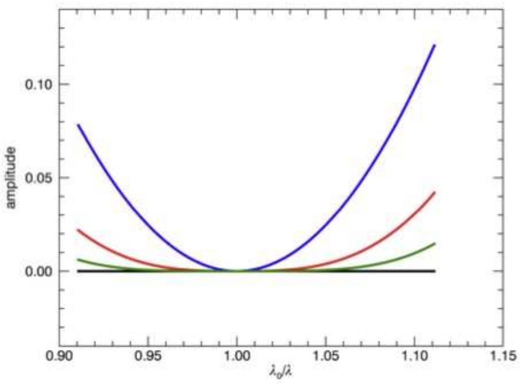

Figure 5.: *The evolution of the nulling factor vs λ for an interferometer equipped with an achromatic chessboard phase-shifter of increasing order.*

It is the proper distribution of *opd* that is responsible of the quasi-achromatization. Why is it so ? Let's define $z = (-1)^{\lambda_o/\lambda} = \exp(j\pi\lambda_o/\lambda)$, so that the cell with $opd = k\lambda_o/2$ produces a complex amplitude z^k. For a basic Bracewell (a unique cell per chessboard), the amplitude is simply $\Lambda = 1 + z$. In other words, $\lambda = \lambda_o$ induces a root of order one on Λ. To obtain a flat Λ around λ_o, let's consider a multiple root : $\Lambda = (1 + z)^n$, so that the higher n, the flatter the nulling vs λ around λ_o, which is exactly the property looked for. Now let's look at the distribution of phase shifts produced by a device that would give this amplitude Λ by developing $(1 + z)^n$ using the binomial coefficients: for instance, when n=3, we get $(1+z)^3 = 1+3z+3z^2+z^3 = 1+z+z+z+z^2+z^2+z^2+z^3$. Let's associate each term with a cell of a double chessboard, such that there are 1 cell of *opd* 0 (z^0), 3 of $\lambda_o/2$ (z^1), 3 of $2\lambda_o/2$ (z^2) and 1 of $3\lambda_o/2$ (z^3). In addition, the odd cells (z^{2k+1}) are grouped on one chess board and the even ones (z^{2k}) on the second one. The trick is here: when $\lambda = \lambda_o$, each cell produces indeed a 0 or a π phase shift as in the classical Bracewell's interferometer, but modulo 2π; this makes the difference. Fig. 6 gives an idea of how chessboards with $n = 3$ or 11 would look.

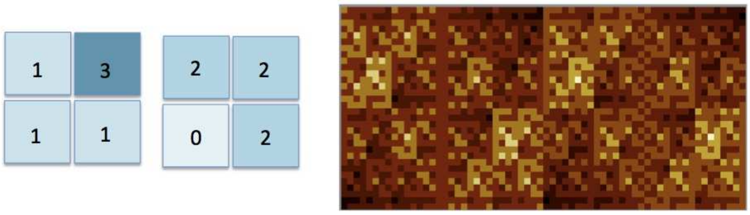

Figure 6.: *Left: the appearance of a chessboard of first order (n=3); right: of order 5 (n=11). The colors code the phase shift and the numbers on left gives the phase shift in unit of* π.

As regards, the x,y distribution of the cells on the chessboard, there is an optimum configuration that produces the best rejection of light around the optical axis, in about the same way as presented for the *deep nulling interferometer*: it makes use of the diophantine relation established by Prouhet. Let's call P_r and Q_r the physical arrangement of the phase shifters at order r. The cells are placed in such a way that $P_r - Q_r$ is a finite difference differential operator of high order. One can achieve this goal with the following iterative arrangement (Pelat et al. 2010): $P_{r+1} = \begin{pmatrix} Q_r + 1 & P_r + 2 \\ P_r & Q_r + 1 \end{pmatrix}, Q_{r+1} = \begin{pmatrix} P_r + 1 & Q_r + 2 \\ Q_r & P_r + 1 \end{pmatrix}$ The theoretical estimate of the absolute maximum bandpass ($n = \infty$) is $\Delta\lambda = 2/3\lambda_o - 2\lambda_o$, i.e. a factor 3 in λ. For a reasonable value $n = 13$, $\Delta\lambda = 0.65\lambda_o - 1.3\lambda_o$ which covers one octave. This means for instance that the Darwin specs (6 – 18 μm) are reachable with two components.

4. DAMNED, the experimental demonstrator of the achromatic chessboard

DAMNED (Dual Achromatic Mask Nulling Experimental Demonstrator) is the experimental demonstrator we developed. The choice of the visible range was dictated by cost and simplicity: on-the-shelf components and detectors, no cryogeny, *but* it implies much more severe specifications on the *opd* accuracy. A simple design was adopted (Fig. 7): with 2 off-axis parabolas, a chessboard mask, a single-mode fiber optics, we simulate two contiguous telescopes recombined in a Fizeau scheme.

The single-mode fiber is essential in the Fizeau scheme: it allows to sum globally the anti-symmetric amplitude and thus to make the nulling effective. The measurement is achieved by an x-y scanning with the fiber optics head of the spot at the focus of the 2nd parabola. A nanometric resolution is reached thanks to piezo actuators.

We tested first a transmissive chessboard in amorphous silica, manufactured by GEPI (Observatoire de Paris) using Reactive Ion Etching. It features 2 $\times$ 8 $\times$ 8 cells of 600 μm size. We obtained a typical nulling factor of 3–7 10^{-3} for broad-band filters. Quasi-achromatism is indeed obtained since we get 8 10^{-3} in the broad range 460-840 nm . The medium nulling performance does agree with

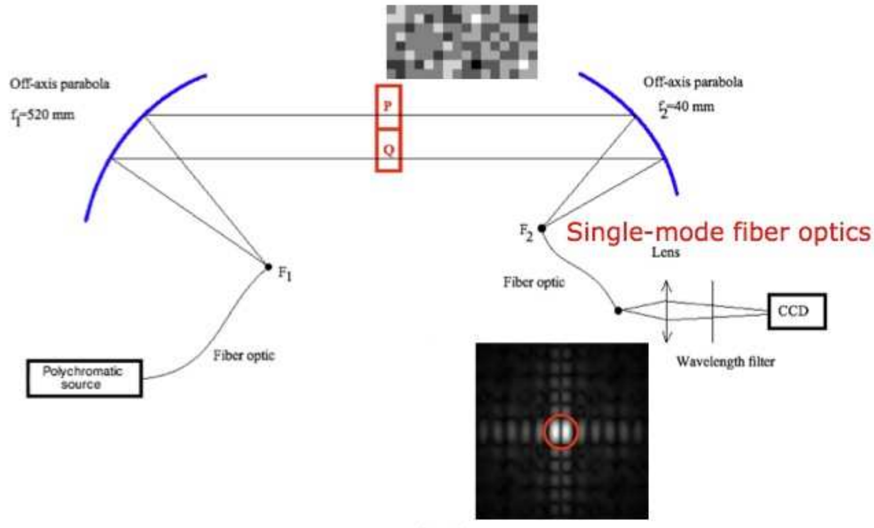

Figure 7.: *Schematics of DAMNED, the laboratory demonstrator.*

numerical simulations using the actual mask cell's thickness: we conclude that performances are limited by the accuracy of steps between cells.

The need for a better accuracy pushed us to change from bulk optics to a deformable mirror controlled in piston. The assets of a phase chessboard synthesized by a segmented deformable mirror are: a) free choice of the central wavelength; b) fine control of each cell's opd; c) versatile way to change the XY distribution; d) open the door to modulation to subtract systematics. Our choice was a segmented Boston Micro-Machine 12 × 12 electrostatic mirror. The optical scheme was then adapted to work in reflection.

Control of flatness is done using phase contrast (i.e. high-pass spatial filtering). The reached accuracy is typically 2–3 nm. A step by step procedure to flatten the DM was designed and appeared to be very efficient as illustrated by Fig. 9 The use of the single mode fiber optics was no longer suited because of the larger PSF (the DM surface is twice smaller than the transmissive mask), so that direct images of the "coffee bean" PSF were done. The performance assessment is less accurate but allowed to check the effectiveness of the nulling. Another method was also tested by scanning all the actuators of the DM while maintaining the nulling pattern, with the source being at a fixed λ (laser): this proved to be the clearest demonstration of the achromatic properties, as illustrated on Fig. 10. The performances are similar to the ones of the transmissive chessboard while order is lower (2 × 4×4), a proof that the improvement is real. A nulling of 10^{-2} is obtained from 0.4 to 0.8 μm: the predicted quasi-achromatism is effectives!

5. Work in progress and conclusion

Present work : we are working on the improvement of the control of the piston accuracy by analyzing the image obtained on several steps; the optical setup is

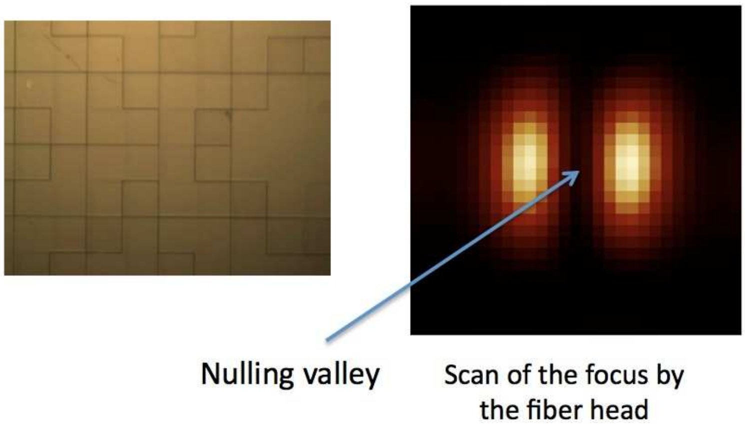

Figure 8.: *Left: macro photo of the transmissive chessboard mask. Right: the result of a x,y scan by the single-mode fiber optics at the focus of the second parabola: the characteristic shape of a coffee bean is observed.*

also the object of some tuning and we are implementing spectroscopy for assessing chromatic performance in a one shot.

Future work : we plan to implement modulation between different nulling configurations to measure possible biases.

If we extrapolate the performance to mid-IR, by assuming that the accuracy on piston is the same at 10 μm, then a null depth of 2 10^{-6} would be reached on a rather low order ($2 \times 8 \times 8$) chessboard: this is within the Darwin specifications!

In conclusion, we think that we have shown that *diophantine optics* is indeed another way of thinking problems in optics. In some cases it may bring a genuine solution to actual problems. For instance, the need for achromatic π phase in nulling interferometry can be solved in an elegant way (one component, symmetry of the arms) thanks to diophantine optics. Using the DAMNED bench, the principle of the achromatic chessboard phase shifter was demonstrated in the lab, both in reflection and transmission. The performances and the mode of operation using a segmented deformable mirror showed that this is a better solution than bulk optics, because it allows to improve the nulling through an active loop. Further developments are currently done to make another step in performances.

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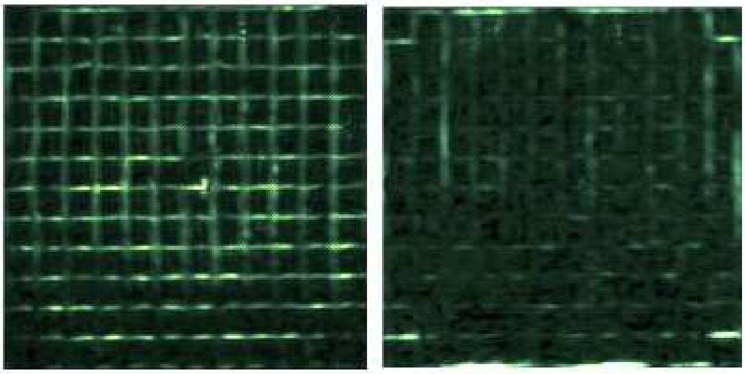

Figure 9.: *The deformable mirror surface before (left) and after (right) flattening.*

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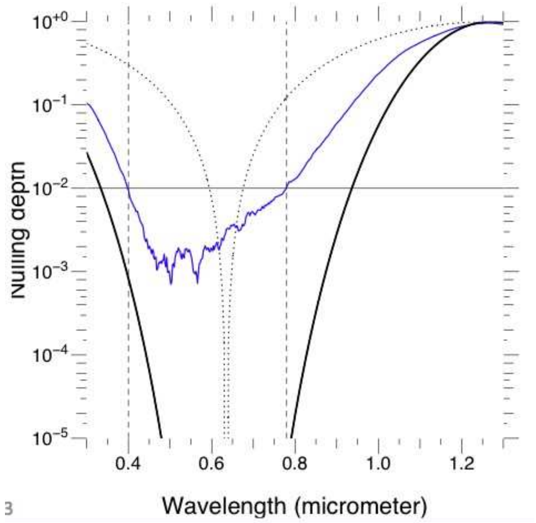

Figure 10.: *The nulling performances vs λ of an achromatic chessboard phase shifter based on a 12 $\times$ 12 segmented deformable mirror. Dotted line: case of a classical Bracewell's configuration, blue thin line:the actual performances with a 2 $\times$ 4$\times$4 pattern, black thick line: the performance of a perfect 2 $\times$ 4$\times$4 mask.*