Space-time positioning at the quantum limit with optical frequency combs

Workshop OHP
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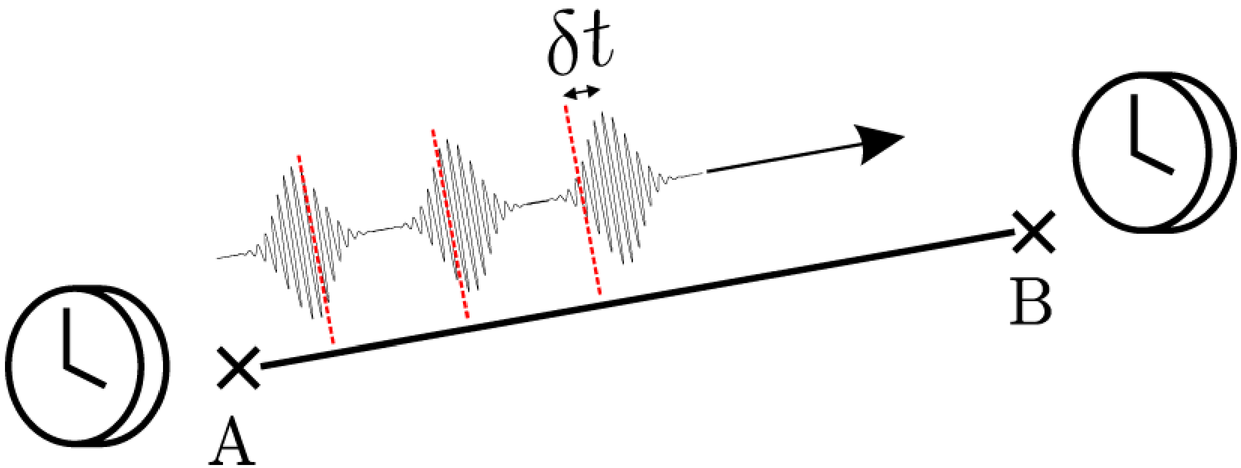


Ranging or **clock synchronization** protocols:
2 observers exchange regularly emitted light pulses

Precision = sensitivity in the estimation of the delay

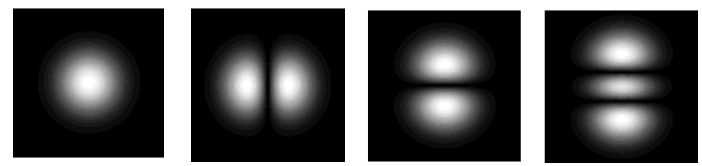
Lamine, Fabre & Treps, PRL **101,** 123601 (2008)

The notion of modes

Mode = solution of Maxwell's equations that contains information on the electric field.

$$\mathbf{E}(\mathbf{r},t)=\sum_{m,n,p} A_{mnp}u_m(\mathbf{r})\,v_n(t)\,\boldsymbol{\epsilon}_p\,e^{-i\omega_0 t}$$

1. Spatial (transverse) modes :



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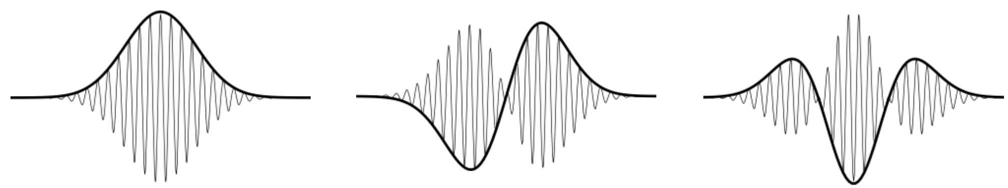
2. Temporal (longitudinal) modes :



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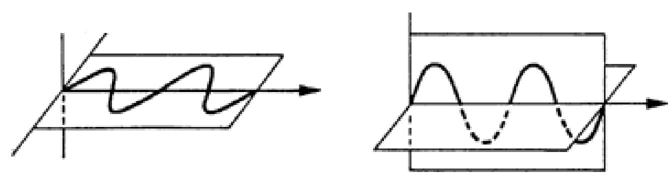
3. Polarization modes :



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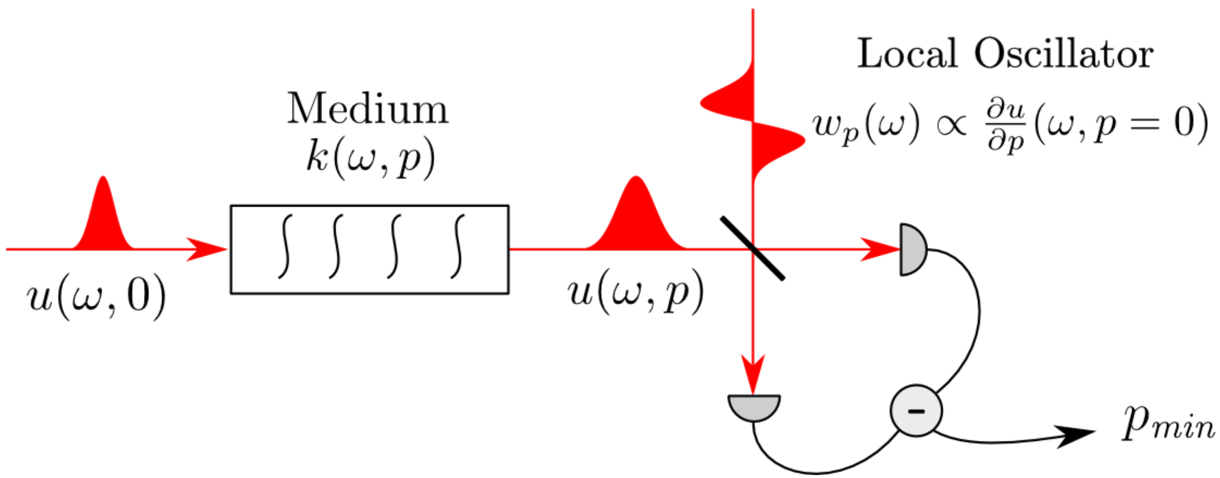
Detection protocol : projective measurements

Measurement strategy for estimating *p:*

Balanced homodyne detection scheme



Detection mode

$$p_{min} = \frac{1}{2\sqrt{N_p}\left\| \left.\frac{\partial u}{\partial p}\right|_{p=0} \right\|}$$

$$u(\omega, p) \approx u(\omega, 0) + p \frac{\partial u}{\partial p}(\omega, p = 0)$$

Signal-to-noise ratio of 1

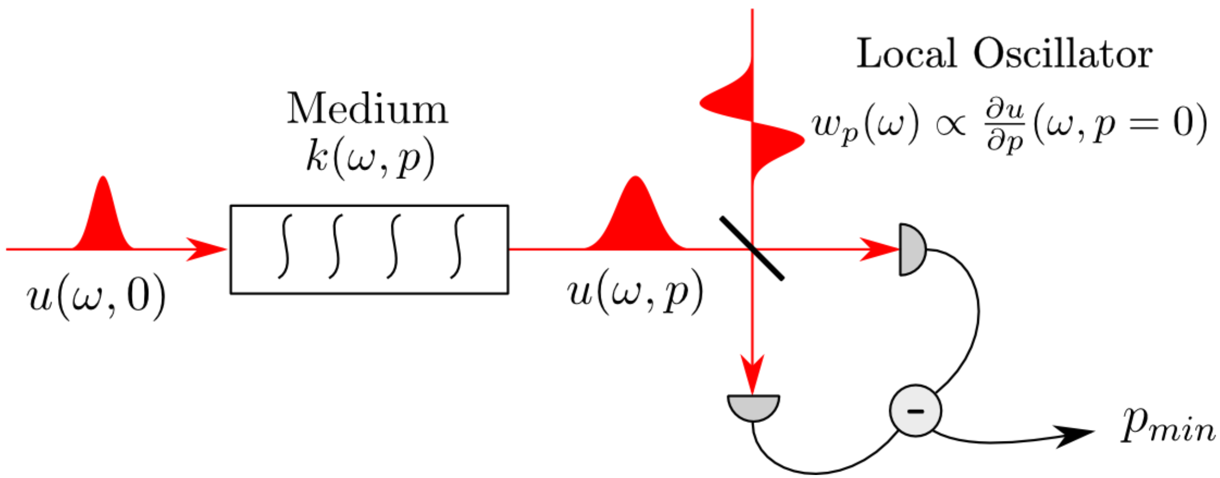
This corresponds to the limit in the parameter estimation theory for **coherent light** = **Standard quantum limit** (SQL) for the estimation of *p*

Pinel *et al.*, PRA **85**, 010101 (2012)

Detection protocol : projective measurements

Measurement strategy for estimating *p:*

Balanced homodyne detection scheme



Detection mode

Variance of detection mode

$$p_{min} = \frac{\sigma}{2\sqrt{N_p}\left\| \left.\frac{\partial u}{\partial p}\right|_{p=0} \right\|}$$

$$u(\omega, p) \approx u(\omega, 0) + p\frac{\partial u}{\partial p}(\omega, p=0)$$

Signal-to-noise ratio of 1

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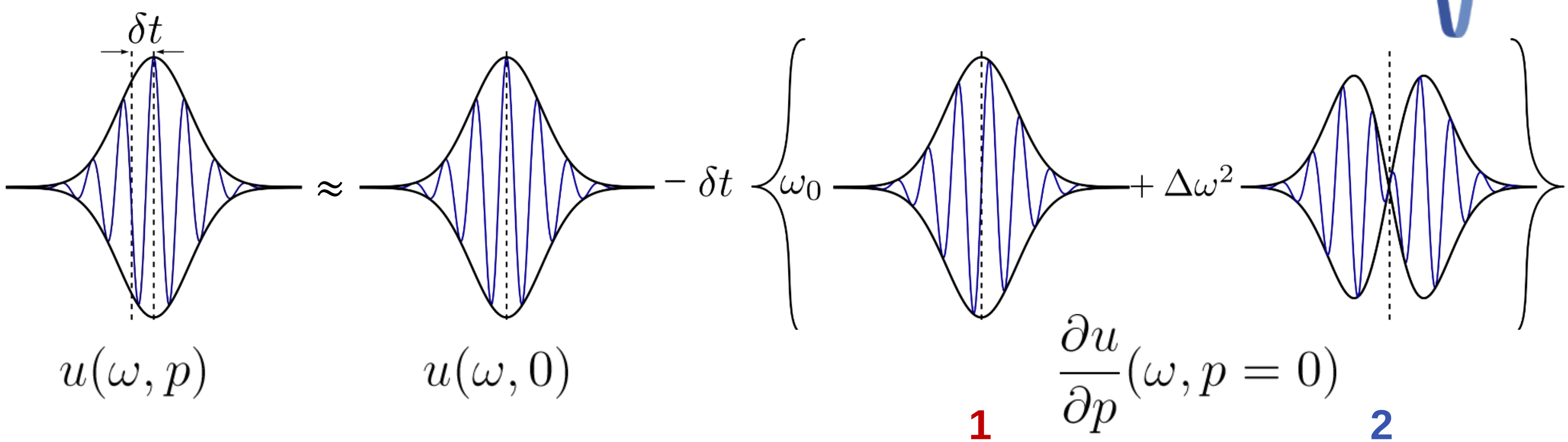
Pinel *et al.*, PRA **85**, 010101 (2012)

Application to ranging

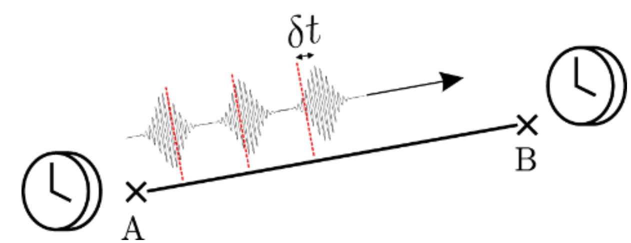


1 2

Detection mode for δt

Estimation of delay δt : 3 projection modes

- **1 :** Carrier displacement : phase velocity mode.
- **2 :** Envelope displacement : group velocity mode.
- **1 + 2 : Detection** mode for δt by linear combination.

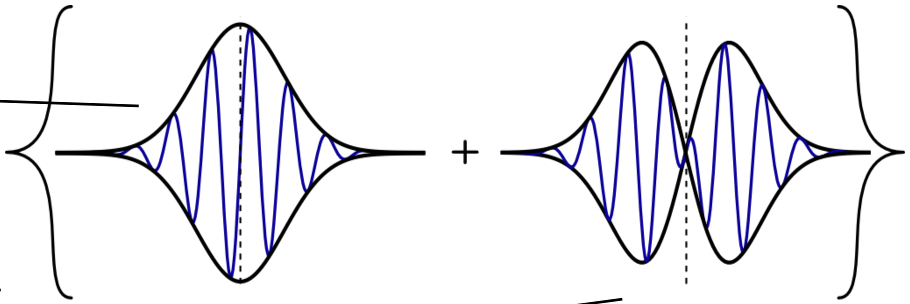


SQL for space-time positioning

Analysis of the two terms:

- coherent interferometric **phase** measurement

$$(\delta t)^{\mathrm{phase}} = \frac{1}{2\sqrt{N_p}\,\omega_0}$$



- incoherent **time-of-flight** measurement

$$(\delta t)^{\mathrm{tof}} = \frac{1}{2\sqrt{N_p}\,\Delta\omega}$$

Measurement with the **detection** mode = **SQL for space-time positioning**

$$(\delta t)_{\mathrm{min}} = \frac{1}{2\sqrt{N_p}\,\sqrt{\omega_0^2 + \Delta\omega^2}}$$

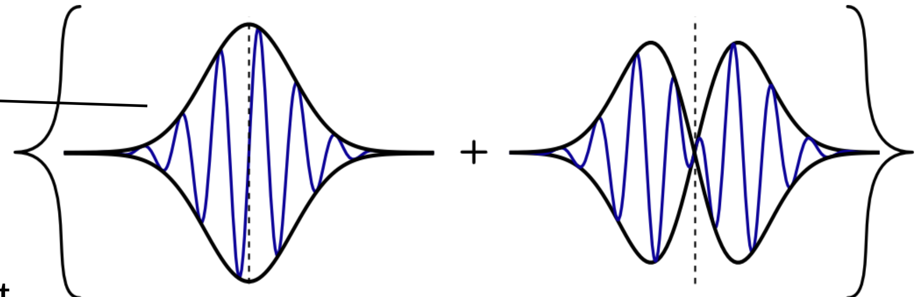
Lamine *et al.*, PRL **101**, 123601 (2008)

SQL for space-time positioning

Analysis of the two terms:

- coherent interferometric **phase** measurement

$$(\delta t)^{\mathrm{phase}} = \frac{\sigma_{phase}}{2\sqrt{N_p}\,\omega_0}$$



- incoherent **time-of-flight** measurement

$$(\delta t)^{\mathrm{tof}} = \frac{\sigma_{tof}}{2\sqrt{N_p}\,\Delta\omega}$$

Measurement with the **detection** mode = **SQL for space-time positioning**

$$(\delta t)_{\mathrm{min}} = \frac{\sigma_{\mathrm{laser}}}{2\sqrt{N_p}\sqrt{\omega_0^2 + \Delta\omega^2}}$$

Lamine *et al.*, PRL **101**, 123601 (2008)

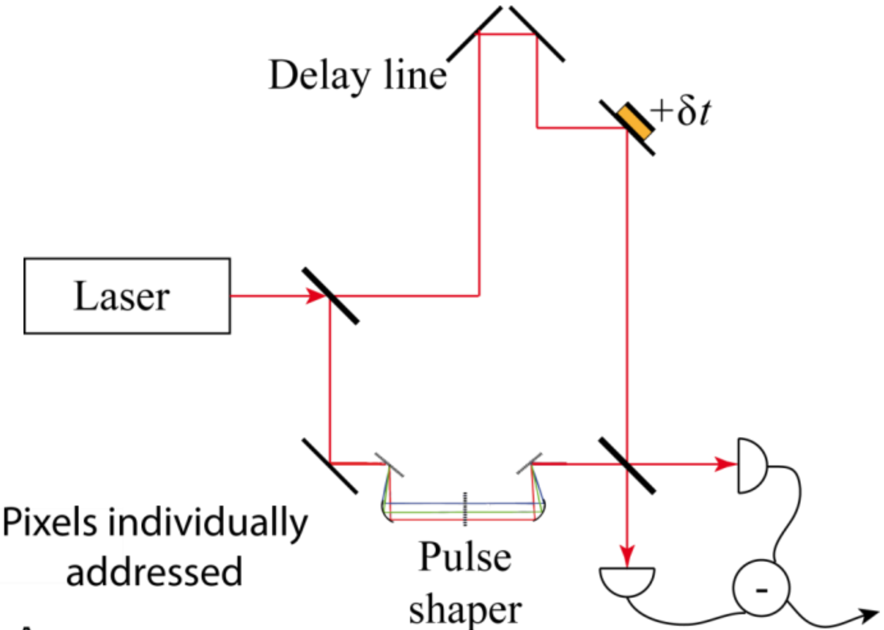
Experimental protocol

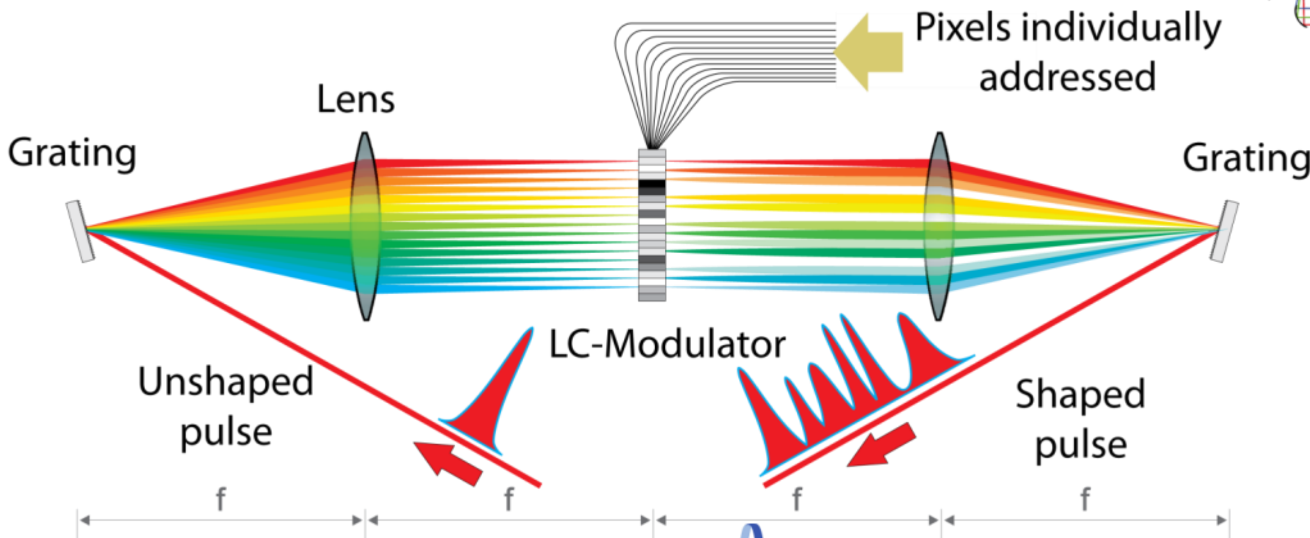
Ti:Sapph laser @ 800 nm, 50 nm FWHM

22 fs pulse @ 156 MHz

At shot noise > ~1 MHz

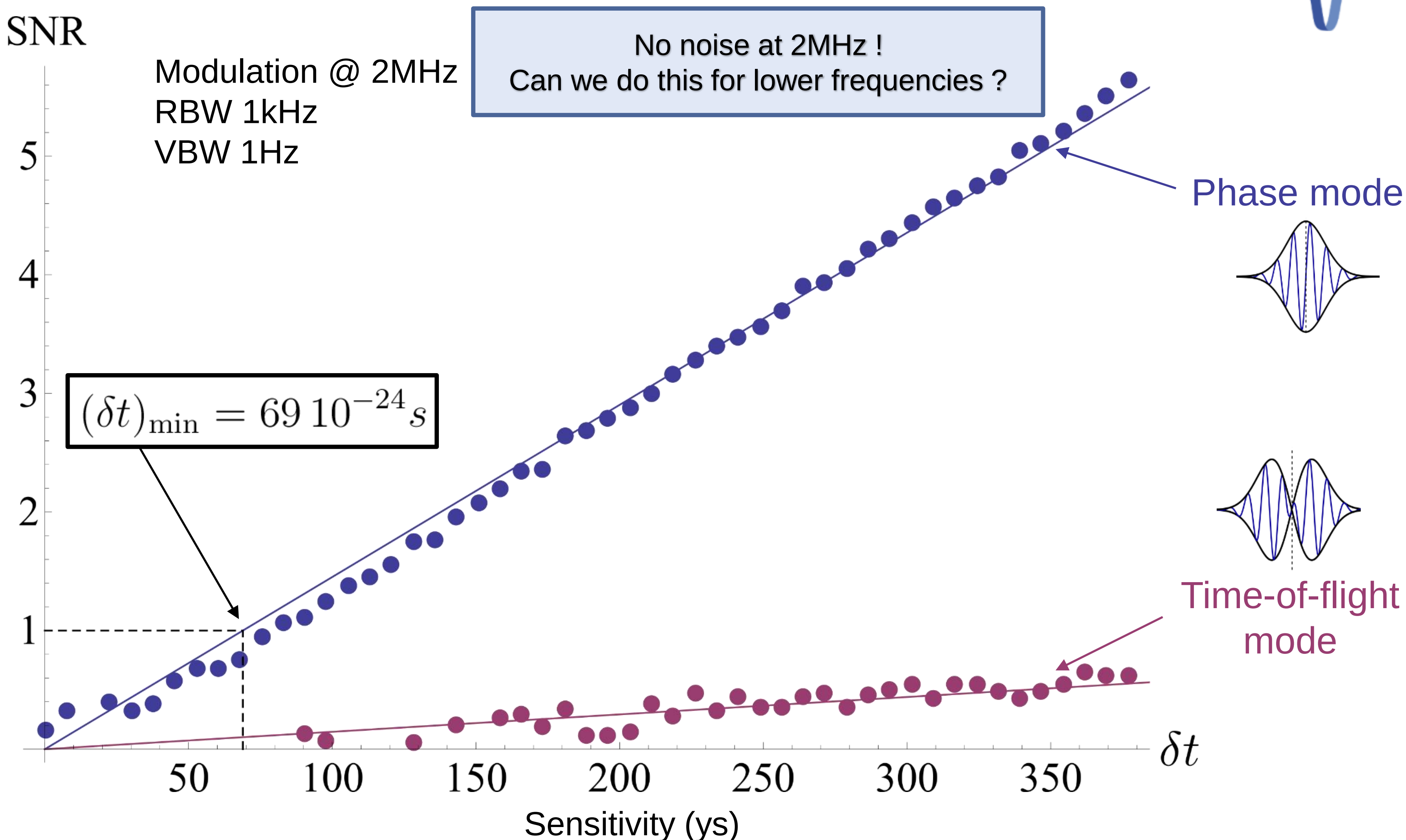
$$(\delta t)_{\min} = \frac{1}{2\sqrt{N_p}\sqrt{\omega_0^2 + \Delta\omega^2}}$$



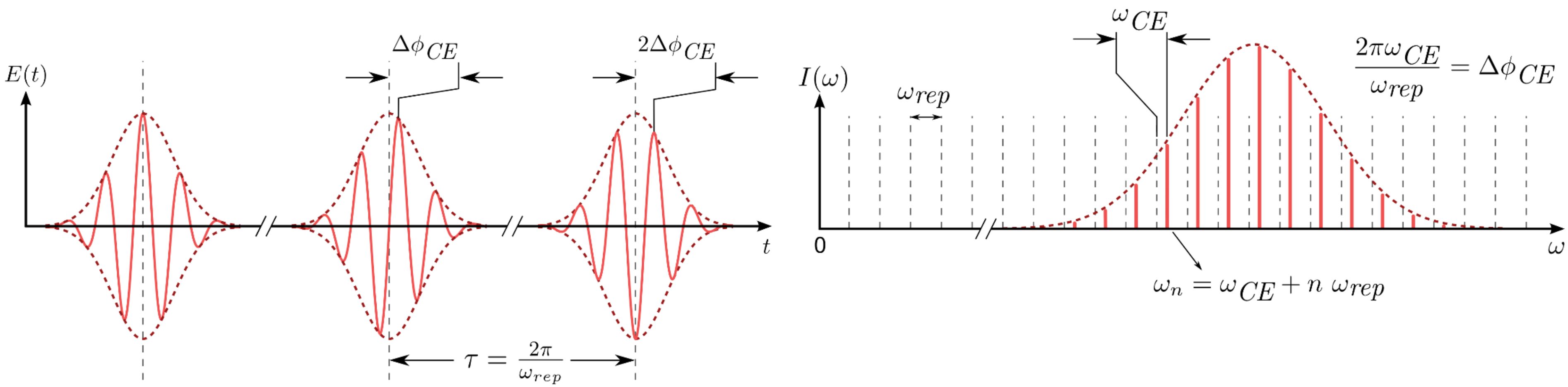


640 pixels
0.2nm / pixel

Results on timing measurement



Noise of a frequency comb



Fluctuation of parameters :

- Output power P
- CEO Frequency f_{CEO}
- Repetition rate f_{rep}
- Spectral center ω_0

Origin of noise :

Spontaneous emission, pump fluctuations, temperature variation, etc…

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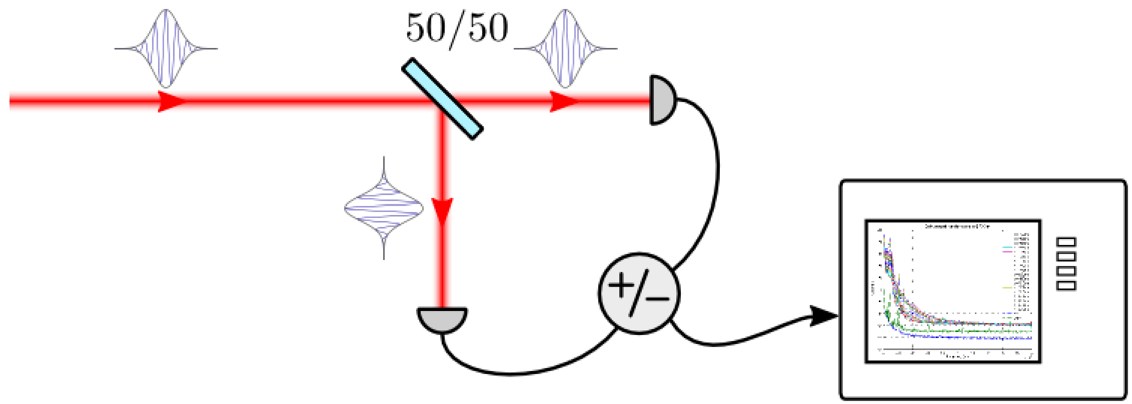
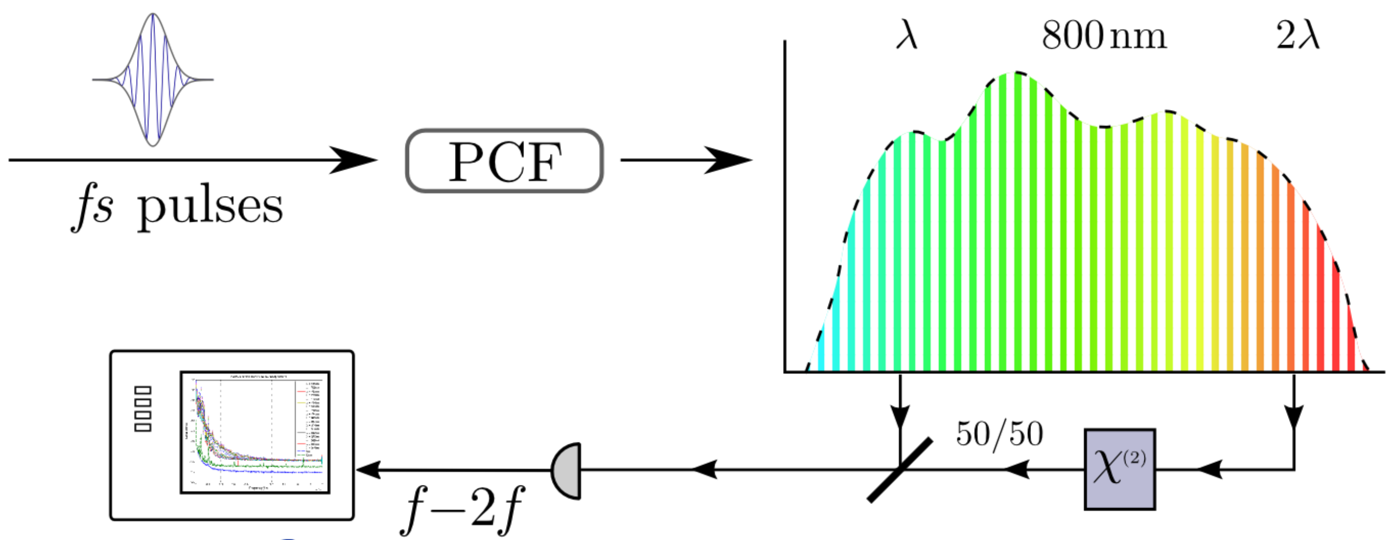
Typical noise of a Ti:Sa Oscillator

- **Intensity** noise :
  balanced detection

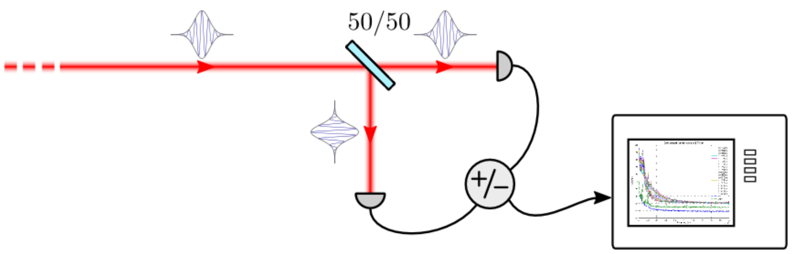


- **Phase** noise :
  standard $f-2f$ interferometer and phase lock loop

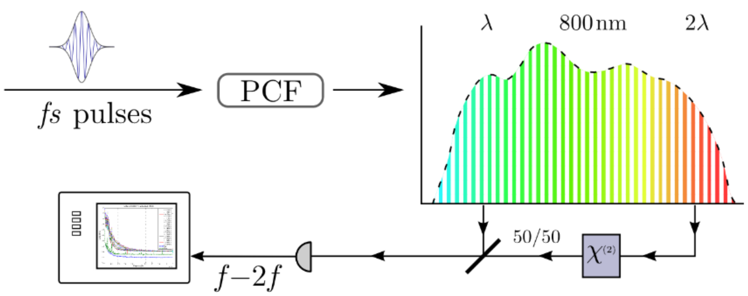


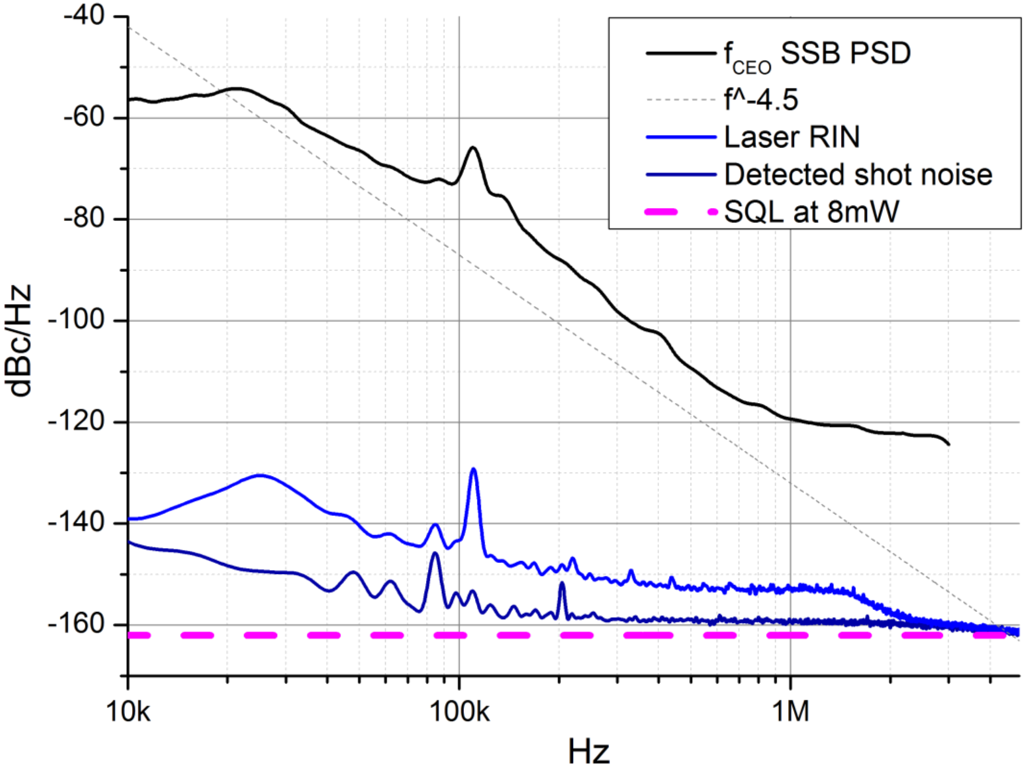
Typical noise of a Ti:Sa Oscillator

- **Intensity** noise : balanced detection



- **Phase** noise : $f - 2f$ interferometer and phase lock loop





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Influence on timing measurement

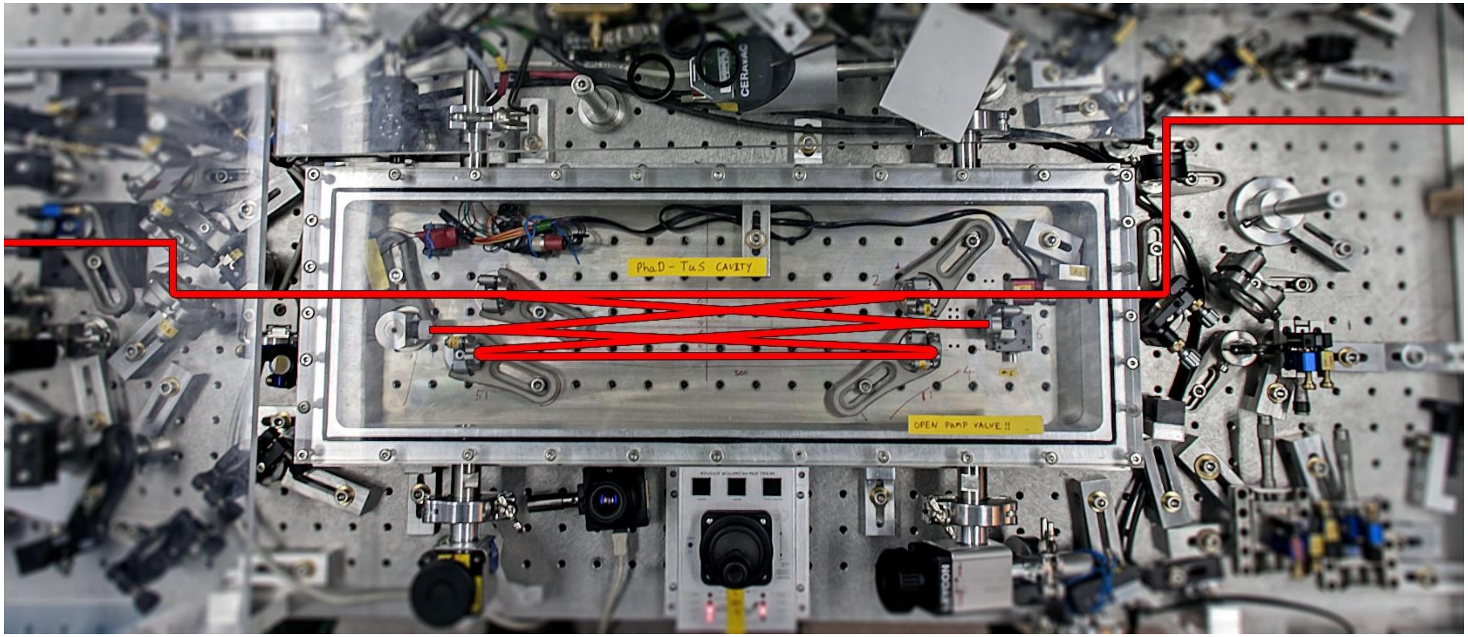
- Sensitivity of detection mode for δt:

$$(\delta t)_{\min} = \frac{1}{2\sqrt{N_p}} \frac{\sqrt{\omega_0^2 \sigma_{\text{phase}}^2 + (\Delta\omega)^2 \sigma_{\text{ampl}}^2}}{\omega_0^2 + (\Delta\omega)^2}$$

Phase noise has the biggest impact on sensitivity

- Solutions :
  - Take measurements at frequencies where noise is at SQL
  - Heterodyning
  - Filtering using a passive cavity

Filtering cavity



Transmissive **high finesse** cavity ($\mathcal{F} = 1200$)

- Low dispersion, resonant over **100 nm** at 50 mbar air pressure
- Passive low-pass filter, cutoff frequency $\boldsymbol{f_c = 120}$ **kHz**
- Contra-propagative **Pound-Drever-Hall** locking scheme

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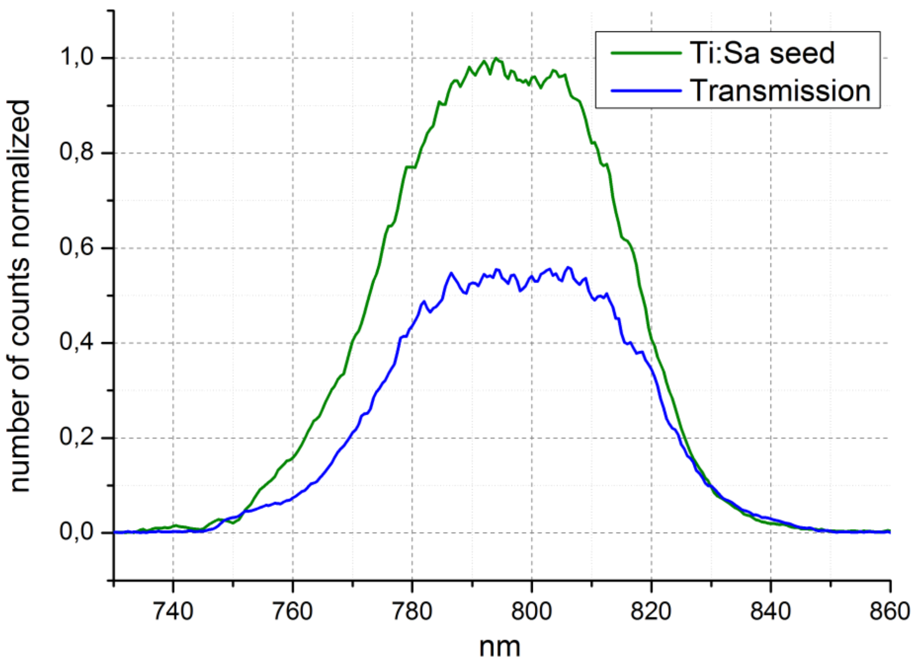
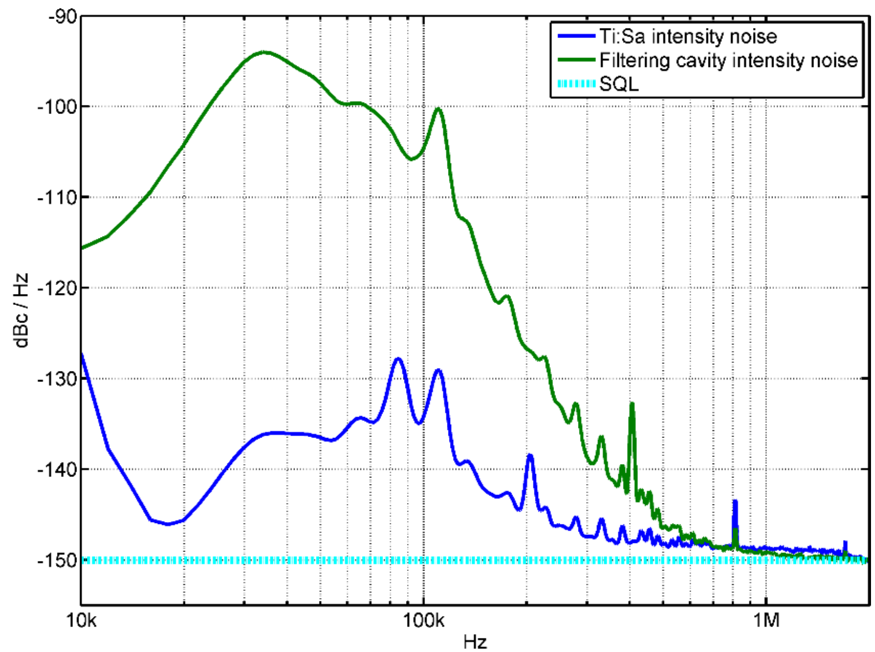
Filtering cavity

Spectral transmission



Amplitude excess noise



Filtering of amplitude noise beyond 800 kHz, but high incident phase noise at low frequencies leads to excess noise by **noise interconversion**

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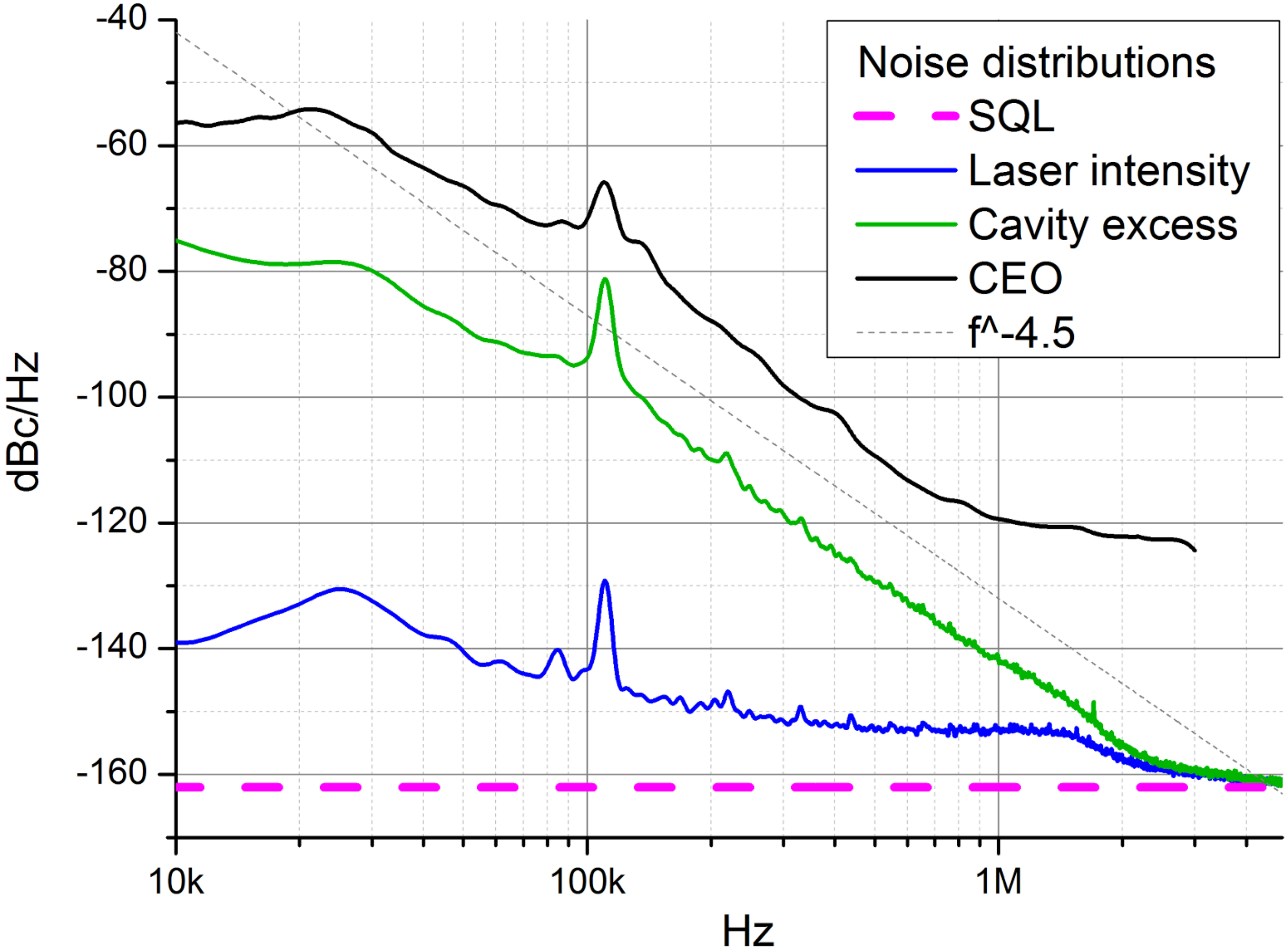
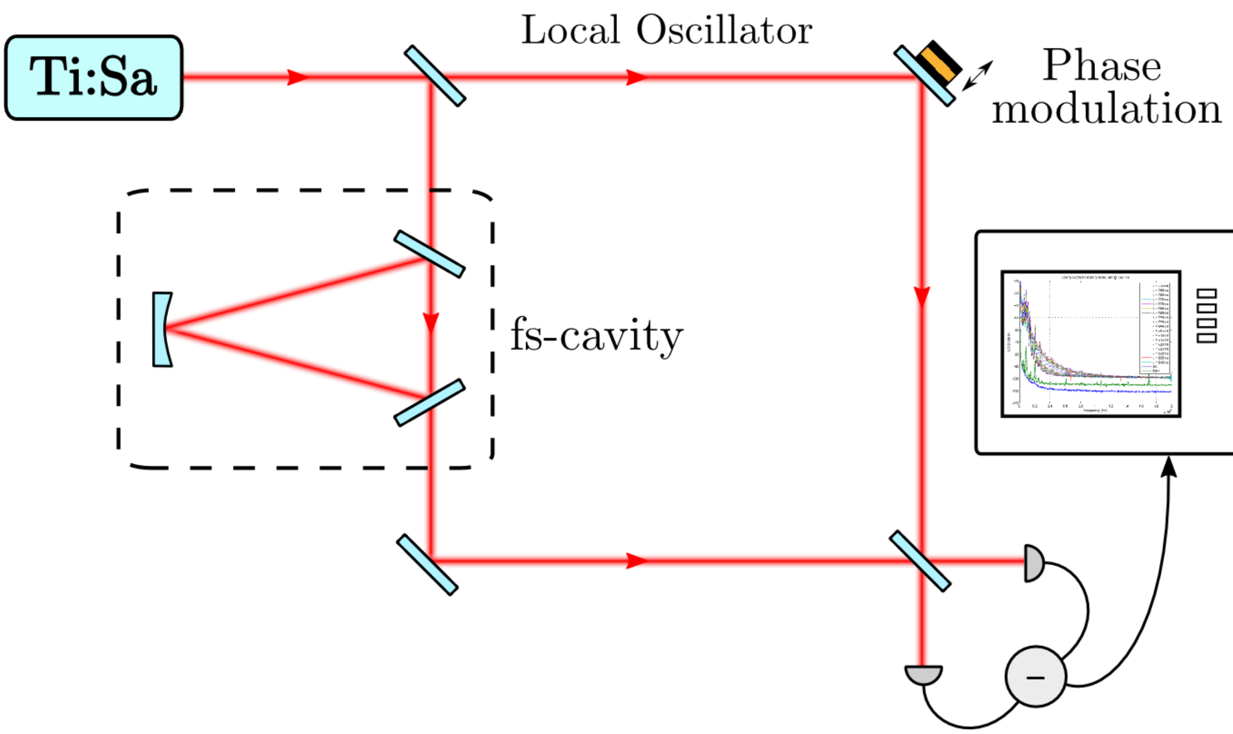
Phase to amplitude noise conversion

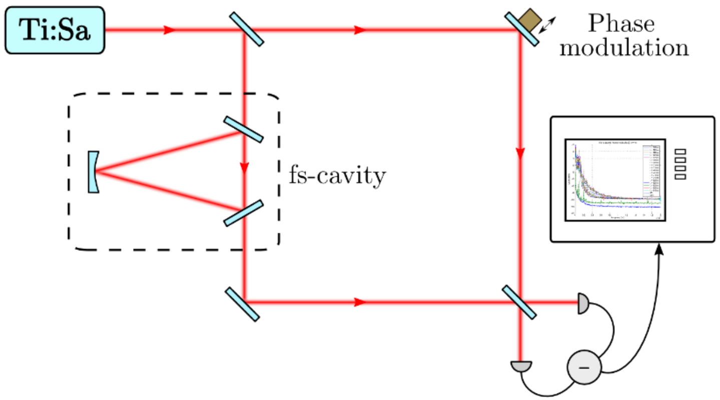


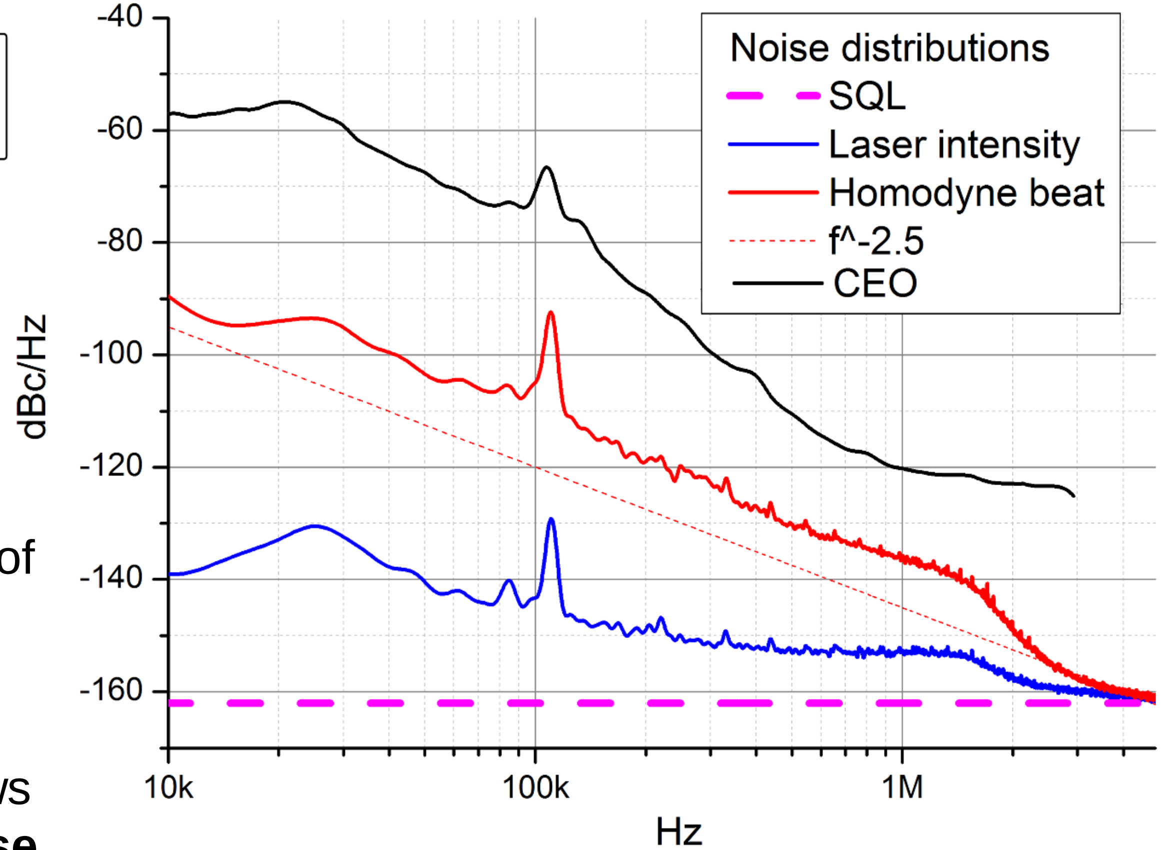
Homodyne detection

- Homodyne detection allows for measurement of **amplitude** or **phase** noise of the signal beam.
- Independent of local oscillator noise



Homodyne detection

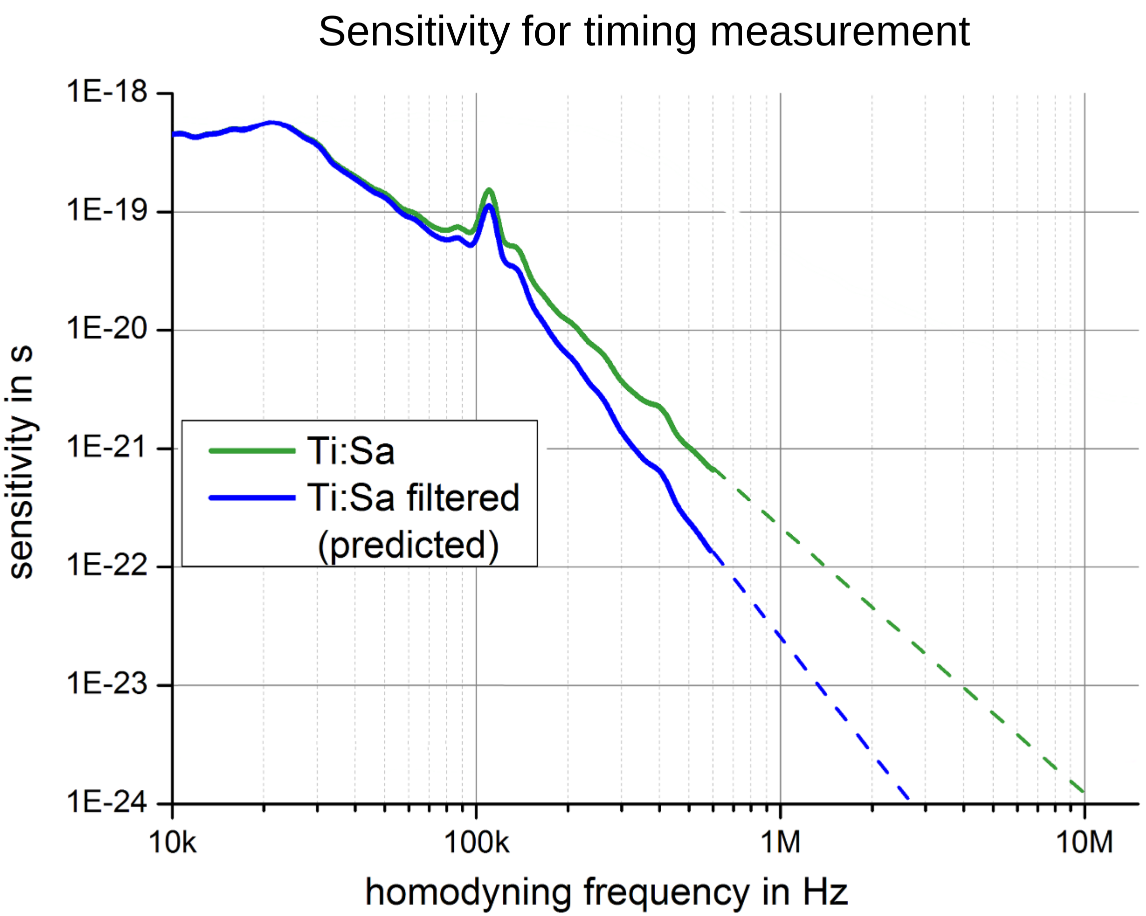




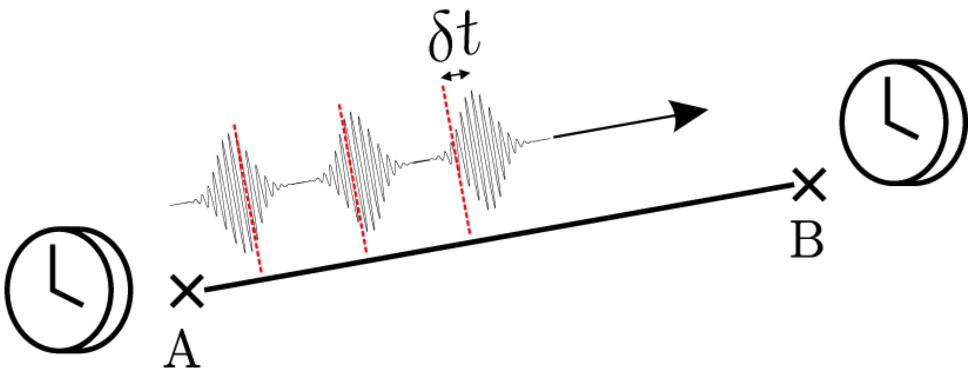
Homodyne detection with a filtering cavity => measurement of **relative phase noise**

Phase quadrature analysis shows **filtering of incident phase noise**

Phase noise filtering for timing experiment



Summary



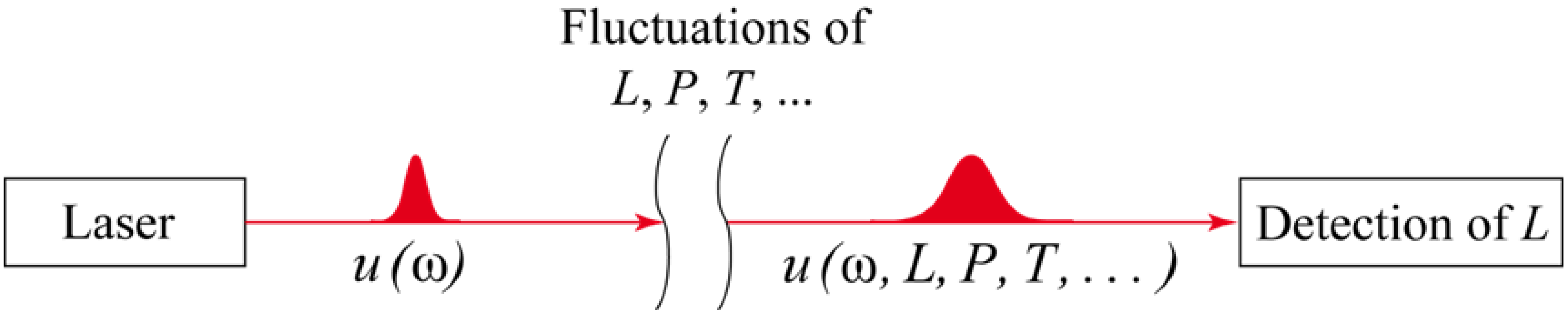
- Projective measurements : different **shaping modes** allow for detection of phase or group velocity by the push of a button
- Possibility to go beyond with **filtering**

What other modes can we construct ? What other parameters can we access?

=> Usefulness for long distance **free-space ranging** experiments ?

Space-time positioning in air

Effect of a dispersive medium on a ranging experiment:



How to remove this parasitic effect?

Find a mode **insensitive** to the environmental parameters

⇔ **orthogonal** to the detection mode of pressure, temperature, etc

Trade-off between **sensitivity** (detection mode) and **accuracy** (purified mode)

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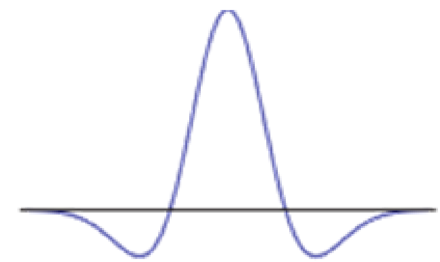
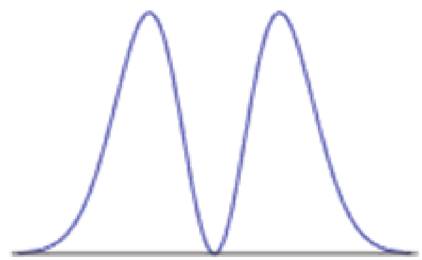
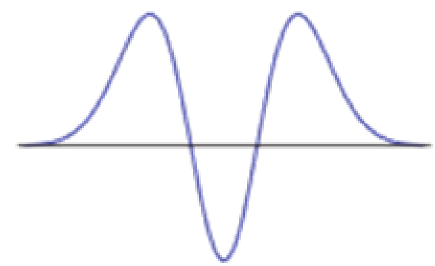
Jian *et al.*, Opt. Exp. **20**, 27133 (2012)

Measuring the phase in dispersive medium

Phase measurement : sensitive to GVD because modes **not orthogonal** !

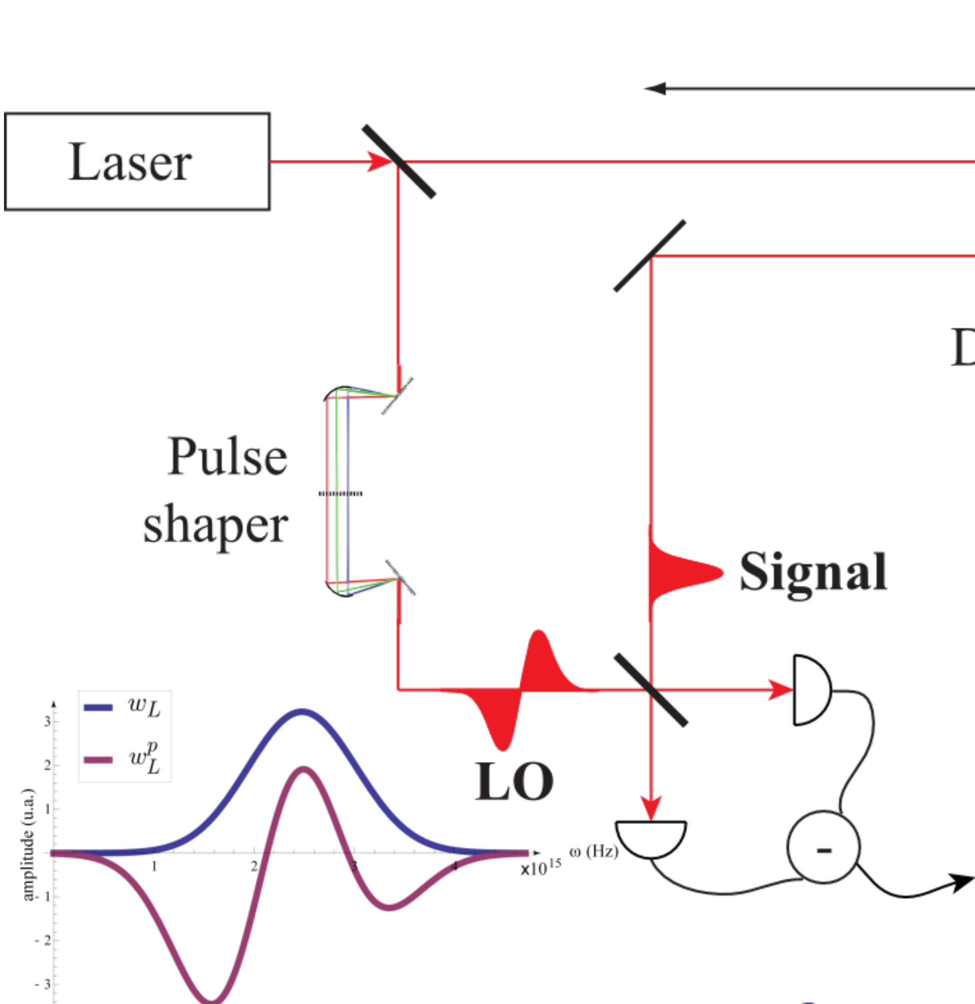
=> Construction of **purified phase mode** that is orthogonal to GVD

=> Phase measurement becomes **insensitive** to GVD, but decrease in **precision**

Parameter	Detection mode	Purified mode
Phase p_ϕ	 w_ϕ	 w_ϕ^p
GVD p_{GVD}	 w_{GVD}	 w_{GVD}^p

Jian *et al.*, Opt. Exp. **20**, 27133 (2012)

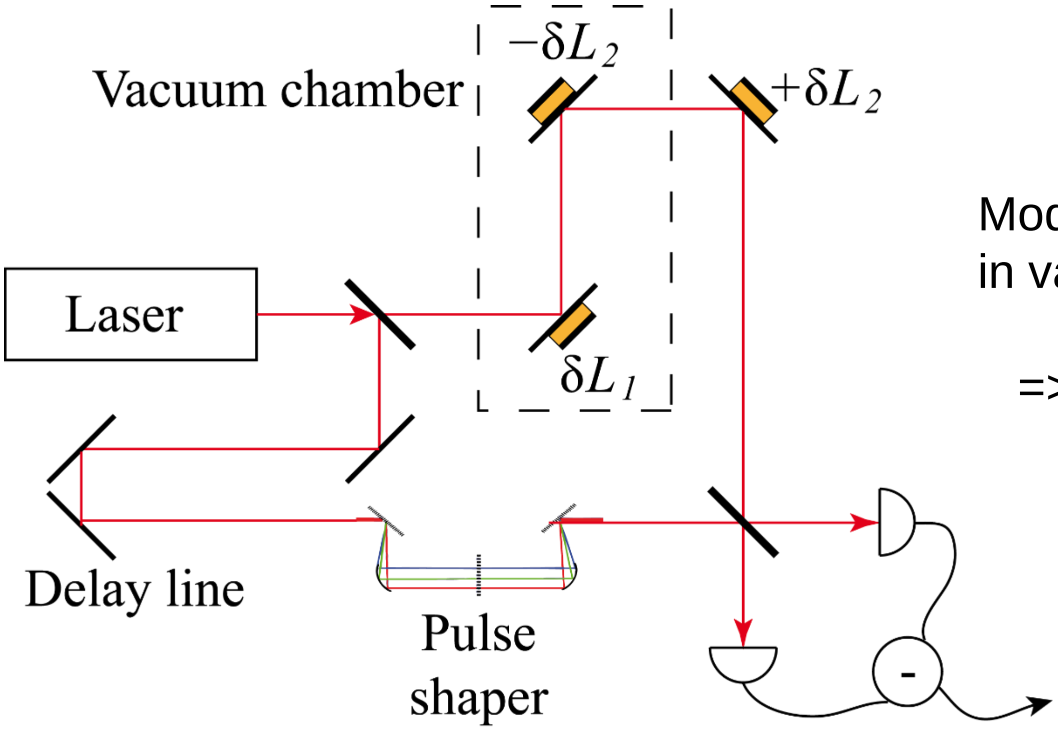
All-optical and real time ranging protocol in air



w_L **Optimal mode**

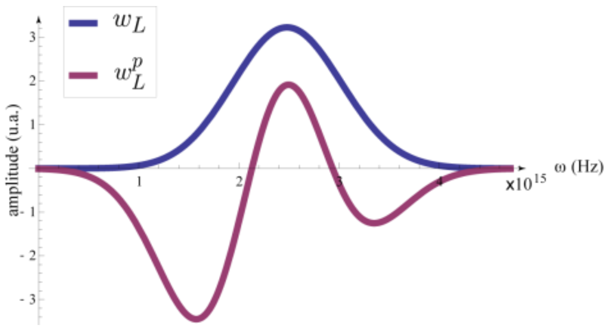
w_L^p **Mode insensitive to environment**

Proposed experiment



Modulation of the distance traveled in vacuum

=> **Effective dispersion modulation**



Conclusion and perspective

A very precise and versatile scheme…

- Measurement of various parameters at the **standard quantum limit**

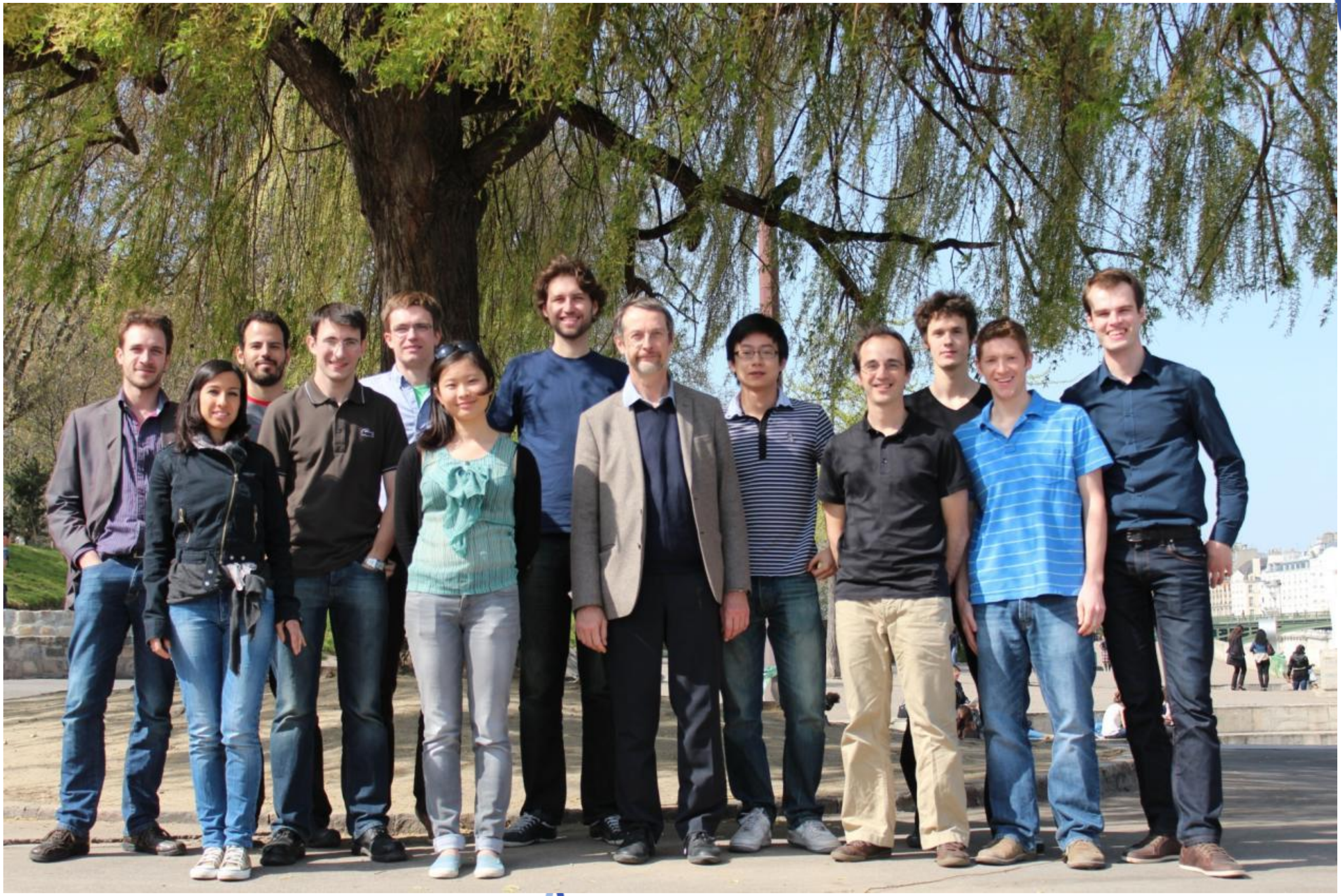
- For ranging, combines **high sensitivity** and **high dynamics**
- In dispersive medium, **no post-processing** needed

- Same scheme for a large range of parameters thanks to **pulse shaping**

… and beyond…

- Possibility to enhance sensitivity by using quantum resources:
  measurement **below the standard quantum limit** becomes possible with **squeezing**

Thank you for your attention



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