Dissipation in resonant systems: Implications of observed orbital configurations

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Resonant/near resonant systems

- What is a resonance between 2 planets?
  - $P_2/P_1 = p/q$ ($p$, $q$ integers)
  - Example: 2/1

- Resonant or near resonant system?
  Resonance width depends on $m_i$, $e_i$
Kepler near-resonant planets

- Distribution of period ratio in Kepler data

- Peaks at resonances $\rightarrow$ convergent migration ($P_2/P_1 \searrow$)

- Peaks slightly shifted to the right $\rightarrow$ tidal dissipation?

(L systems near but outside of resonances)


Lissauer et al. (2011), Fabrycky et al. (2014)
Formation scenario

Convergent migration (in protoplanetary disk)

Evolution under tides (slow)

Capture in resonance

End of migration

$t$

$P_2 / P_1$

0.0
0.1
0.2
0.3
0.4
0.5
2.0
2.1
2.2
2.3
2.4
2.5

$P_2$

$P_1$
Other possible explanations for the shift:

- protoplanetary disk - planets interactions
  Rein (2012), Baruteau & Papaloizou (2013)

- planetesimals - planets interactions
  Chatterjee & Ford (2015)

- in-situ formation of planets
  Petrovitch, Malhotra, Tremaine (2013), Xie (2014)
Why tidal dissipation?

- Distribution of period ratio close to resonances (2:1 + 3:2)

![Graph showing CDF of period ratio](image)

Delisle, Laskar (2014)
Why tidal dissipation?

- Distribution of period ratio close to resonances (2:1 + 3:2)

Evidence for tidal dissipation

KS-tests
- Close-in vs Farthest: 0.08%
- Close-in vs Intermediate: 3.5%
- Intermediate vs Farthest: 10%

Delisle, Laskar (2014)
Analytical model of resonances

- First order resonances (2/1, 3/2, etc.)
  
  Integrable approximation is straightforward
  

- Higher order resonances (3/1, 5/2, etc.)
  
  2 degrees of freedom (not integrable)
  
  New simplifying assumption
  
  $e_1/e_2 \approx (e_1/e_2)_{\text{forced}}$ (ecc. ratio at resonance center)

  Integrable pendulum-like approx.

  $H = -(I - \delta)^2 + 2R \cos(q\theta)$

  Delisle, Laskar, Correia, Boué (2012)
  Delisle, Laskar, Correia (2014)
Dissipative evolution in resonance

- Dissipation affects the resonant motion in 2 ways

- Relative amplitude: $A = \frac{\text{Amplitude}}{\text{Width}}$
  - if $A \downarrow$ Locked in resonance, $P_2/P_1 \approx p/q$
  - if $A \uparrow$ Escape from resonance, $P_2/P_1$ no more locked

\[ 2\lambda_2 - \lambda_1 \]

\[ P_2/P_1 \]

Width change
Spiraling of trajectory
Migration in protoplanetary disk

- $A \nearrow$ (unstable res.) $\iff \frac{T_{e,1}}{T_{e,2}} < \left(\frac{e_1}{e_2}\right)_{forced}^2$

  ecc. damping timescales
  (by disk-planet interactions)

→ Escape with $P_2/P_1 \searrow$ (convergent migration)

Delisle, Correia, Laskar (2015)
Migration in protoplanetary disk

- $A \uparrow$ (unstable res.) $\iff \frac{T_{e,1}}{T_{e,2}} < \left( \frac{e_1}{e_2} \right)^2_{forced}$

  ecc. damping timescales (by disk-planet interactions)

  $\rightarrow$ Escape with $P_2/P_1 \downarrow$ (convergent migration)

- Observed resonant systems
  $\rightarrow$ constraints on disk properties
  (ex: aspect ratio, surface density profile...)

Delisle, Correia, Laskar (2015)
Constraints on disk properties

Varying disk properties

\[ \tau = +\infty, \; K = 70 \]
\[ \tau = 10, \; K = 34 \]
\[ \tau = 8, \; K = 30 \]
\[ \tau = 4, \; K = 17 \]

Varying disk properties

\[ \frac{P_2}{P_1} \]

\[ \frac{e_1}{e_2} \]

\[ \angle_1 - \angle_2 \]
\[ 3\angle_1 - \angle_1 - 2\angle_2 \]

ex: HD 60532 b, c
Observed in 3/1 res.
→ Did not escape
→ Constraints on disk
(aspect ratio...)

Delisle, Correia, Laskar (2015)
Tidal dissipation

\[ \tau = \frac{T_1}{T_2} \]
\[ \tau_c \approx L \left( \frac{e_1}{e_2} \right)^2 \frac{4 + |k_2|(1+L)}{4L-|k_1|(1+L)} \]
\[ \tau_\alpha = \left( \frac{e_1}{e_2} \right)^2 \]
\[ L \approx \frac{m_1}{m_2} \left| \frac{k_1}{k_2} \right|^{1/3} \]

- **\( \tau < \tau_c \):** Amplitude ↗ → separatrix crossing possible
  - **\( \tau < \tau_\alpha \):** Diverging
  - **\( \tau > \tau_\alpha \):** Converging

- **\( \tau > \tau_c \):** Amplitude ↘ → evolution close to libration center
  - **\( q = 1 \):** Diverging
  - **\( q > 1 \):** Staying in resonance

Delisle, Laskar, Correia (2014)
Constraints on planets nature

ex: GJ 163

<table>
<thead>
<tr>
<th>Parameter</th>
<th>[unity]</th>
<th>b</th>
<th>c</th>
<th>d</th>
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- Planets b, c close to 3:1 MMR (order 2)

\[
\frac{P_2}{P_1} = 2.97 < 3 \quad \text{Internal circulation (converging)}
\]

\[ \tau_\alpha < \tau < \tau_c \]

Delisle, Laskar, Correia (2014)
Constraints on planets nature

\[ \frac{\Delta t_2}{\Delta t_1} \]

GJ 163b, c are here
GJ 163b: gaz
GJ 163c: rock

Delisle, Laskar, Correia (2014)
Conclusion

• Classification of outcome of dissipative process in resonance

• Constraints on systems properties from period ratio
  – Disk properties (disk-planet interactions)
  – Planets nature (tidal dissipation efficiency)

• Analytical model
  – Better understanding of these complex process
  – First approximation of constraints
  – Need numerical simulations for precise constraints